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# B.Sc. III Paper DSE- E09 (Mathematical analysis)

# Question Bank

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## Q. Multiple Choice Questions.

1) Integral $\int_a^b f(x)\,dx$ is said to be improper if … … ….
   a) both the limits are finite b) f(x) is bounded c) one or both integration limits are infinite d) none of these
2) If range of integration is finite and f(x) is bounded then $\int_a^b f(x)\,dx$ is called … … ….
   a) proper integral b) improper integral c) double integral d) none of these
3) $\int_0^\infty \sin x\,dx$ is improper integral of … … …
   a) first kind b) second kind c) third kind d) not improper integral
4) If in $\int_a^b f(x)\,dx$ integration limits are infinite and f(x) is unbounded then it is an improper integral of .......
   a) first kind b) Second kind c) Third kind d) none of these
5) $\int_1^\infty \frac{dx}{x}$ is … … …
   a) convergent b) divergent c) oscillatory d) none of these
6) Integral $\int_a^\infty \frac{dx}{x^n}$ , $a > 0$ *is convergent if* … … ….
   a) n = 1 b) n < 1 c) n > 1 d) never convergent
7) $\int_0^\infty \frac{\sin mx}{a^2+x^2}\,dx$ is … … ..
   a) converges but not absolutely b) absolutely convergent c) divergent d) none of these
8) If $\int_a^\infty f(x)\,dx$ converges and phi(x) is bounded and monotonicfor x > a, then $\int_a^b f(x)\,dx$ is convergent is ........... test.
   a) Dirichlet's b) comparison c) Abel's d) none of these
   b)
9) Every open set of real number is union of …….
   a) countable union of open interval b) countable union of closed interval
   c) finite intersection of open interval d) finite union of open interval

10) The set S = [0, 1] has …….

   a) maximal element b) minimal element only

   c) both minimal and maximal d) none of these

11) The set of natural numbers has ……

   a) upper bound b) lower bound c) maximal element d) none of these

12) Each lower Darboux's sum represents area of union of rectangles ………… the origin.

a) outsides b) forms c)insides d) none of these

13) U (f, p) is the sum of circumscribed ……..

a) plane b) square c) circle d) rectangles

14) Upper Darboux integral is not equal to Lower Darboux integral implies that……. f is not integral

a) f is not integrable b) f is integrable c) may or may not be integrable d) none of these

15) If f is bounded function on closed and bounded interval [a, b], f is said to be Riemann integrable on [a, b] if …….

a) $\int_a^{b^-} f = \int_{a^-}^{b} f = \int_a^b f$ b) $\int_a^{b^-} f \neq \int_{a^-}^{b} f = \int_a^b f$ c) $\int_a^{b^-} f = \int_{a^-}^{b} f \neq \int_a^b f$ d) none of these

16) If f is bounded function on [a, b], and if P and Q are partition of [a, b] then ---------

a) L(f, p) ≥ U (f, p) b) L(f, p) = U (f, p) c) L(f, p) ≤ U (f, p) d)L(f, p) ≠ U (f, p)

17) A bounded function on [a, b] is integrable iff for each $\varepsilon > 0$ there exist partition of

[a, b] such that

a) U [f, p] – L[f, p] > $\varepsilon$ b)U [f, p] – L[f, p] ≤ $\varepsilon$

c) U [f, p] – L[f, p] < $\varepsilon$ d)U [f, p] – L[f, p] = $\varepsilon$

18) The mesh of partition P is ……… of the subinterval comprising P.

a) maximum length b) minimum length c) equal length d) same length

19) Every monotonic function f on [a, b] is ………

a) Differentiable b) Integrable c) Continuous d) none of these

20) A function f is monotonic means either increasing or decreasing

a) increasing b)decreasing

c)either increasing or decreasing d) increasing and decreasing

21) If f and g are integrable on [a, b] then $\int_a^b f \leq \int_a^b g$ provided the condition that

a) g(x) ≤ f(x) b) g(x) = f(x) c)g(x) ≥ f(x) d)g(x) > f(x)

22) In equality, $\int_a^b cf = c\int_a^b f$ is …………

a) complex number b) real number c) rational number d)none of these

23) If f is continuous at $X_0$ in [a, b] then f is differentiable at $X_0$ and ……

a) $F'(X_0) = f(X_0)$ b)$F(X_0) = f(X_0)$ c) $F'(X) = f(X_0)$ d)$F'(X_0) = f(X)$

24) The Fourier series of a periodic function f(x) with period …….

a) $2n\pi$ b) $4\pi$ c)$2\pi + 1$ d) $2\pi$

25) Half range series contains terms of …….

a) sine only b) cosine only c) sine and cosine only d) either sine or cosine

## Q. Long and short Questions.

1) State and prove Darboux's theorem.

2) If f(x) = $\begin{cases} x^2; & if\ x\ is\ rational \\ x^3; & if\ x\ is\ irrantional \end{cases}$

Evaluate the upper and lower integral of f(x) in [0, 2].

3) Prove that, the necessary and sufficient condition for the integrability of a bounded function f is that to every $\epsilon > 0$, there corresponds a division D such that the corresponding oscillatory sum ω (D) < $\epsilon$.

4) Show that the integral function of an integrable function is continuous.

5) Define improper integral and test the convergence of $\int_1^\infty \frac{dx}{x^{\frac{3}{2}}}$.

6) Show that the integral $\int_0^\infty \frac{\sin x}{x}\, e^{-ax}\, dx$, a ≥ 0 is convergent.

7) Test the convergence of $\int_1^\infty \sin x^p\ dx$.

8) Find the Fourier cosine series for the function f(x) = $x^2$ in the range $0 \le x \le \pi$.

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# B.Sc. III Paper DSE- E10 (Abstract Algebra)

# Question Bank

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## Q. Multiple Choice Questions.

1) If G is a group then the subset $\left\{x \in \frac{G}{xa} = ax, for\ all\ a \in G\right\}$ $is\ called \ldots \ldots ..$

a) centre of G b) normalizer of G c) right coset d) left coset

2) A cycle of length two is called ……

a) an even permutation b) transportation c) disjoint permutation d) an Idempotent

3) O(G) = 20 then group G may have subgroup of order ........... a) 3 b) 5 c)7 d) 9

4) conjugacy classes of $S_3$ are .............

a) 0 b) 1 c) 2 d) 3

5) A permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 4 & 3 & 5 & 6 & 2 \end{pmatrix}$ is a cycle of length …....

a) 2 b) 4 c)6 d) 3

6) Any two conjugate subgroups of group are..........

a) Isomorphic b) not Isomorphic c) may or may not Isomorphic d) all of these

7) f = (1 2 4 3) is ……… permutation.

a) odd b) even c) disjoint d) permutation

8) Number of generators of a finite cyclic group of order n is ..........

a) n b) $n^2$ c) 2n d) $\phi(x)$

9) An Imbedding mapping is ....... $\theta\ from\ R to\ R'$.

a) one-one b) onto homomorphism

c) one-one homomorphism d) onto isomorphism

10) A subgroup of cyclic group is…….

a) cyclic b) not cyclic c) symmetric d) not symmetric

11) For Euler's $\emptyset$ function $\emptyset$ (10) =………..

a) 4 b) 5 c) 6 d) 10

12) From the following which is maximal ideal ....

A. 2Z B. 6Z C. 8Z D. 10Z

13) If G = $\{1, -1, \mathrm{i}, -\mathrm{i}\}$ is a group under multiplication operation then O(-i) = .........

a) 1 b) 2 c) 3 d) 4

14) An one-one homomorphism is called ………..

a) epimorphism b) monomorphism c) automorphism d) endomorphism

15) A homomorphism $f: G \to G'$ is one-one iff ……....

a) Kerf =$\{e\}$ b) $Kerf = \{0\}$

c) Kerf = F d) none of these

16) A homomorphism from a group G to itself is called .........

a) Endomorphism b) monomorphism c) automorphism d) epimorphism

17) Degree of constant polynomial is ......

a) 0 b) Not define c) may or may not define d) 1

18) An ideal P of a commutative ring R is prime if and only if $\frac{R}{P}$ is ...........

a) Integral domain b) field c) skew-field d) simple group

19) Every homomorphic image of a group G is isomorphic to ........

a) quotient group of G b) subgroup of G c) symmetric group d) semigroup

20) If R is ring with unity then R= $\{0\}$ is called …......

a) trivial ring b) subring c) non-trivial ring d) integral domain

21) A commutative ring R is called Integral domain if ab= 0 in R $\Longrightarrow$………

a) either a = 0 or b = 0 b) a = 0 and b = 0

c) ab = p d) ab$\neq$ 0

22) Division ring is …......

a) Simple ring b) field c) commutative ring d) Integral domain

23) Let R[x] be ring of polynomials over R then

(i) R is commutative iff R[x] is commutative.

(ii) R has unity then R[x] has unity.

A.Only (i) is true B. Both (i) and (ii) are true
C. Only (ii) true D. Both (i) and (ii) are false

24) Every ideal in a Euclidean domain is......... ideal.

A. Prime B. Not C. Principle D. Maximal

25) Let R be the commutative ring with unity s.t. R[x] is PID, then R is ......

A. PID B. Field C. UFD D. Integral domain

## Q. Long answer type questions.

1) Define a subgroup of G. Prove that a non-empty subset H of group G is a subgroup of G iff a,b $\in$ H implies that $ab^{-1} \in$ H.

2) Define characteristic of Ring. If D is an integral domain then characteristic of D either zero or prime number.

3) Prove that a field is an integral domain.

4) Define conjugacy in group and prove the class equation.

5) State and prove second theorem of isomorphism.

6) Prove that R be a commutative ring with unity, an ideal M of R is maximal ideal of R iff $\frac{R}{M}$ is a field.

7) Define principle ideal domain and show that Z is PID.

8) Define UFD, State and prove Guass theorem

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# B.Sc. III Paper DSE- E11 (Optimization Techniques)

# Question Bank

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## Q. Multiple Choice Questions.

1) The solution to a transportation problem with m-sources and n-destinations is feasible if the number of allocations is ……..

a) m + n  b) mn  c) m – n  d) m + n – 1

2) The allocation cells in the transportation table will be called ……………. cell.

a) Occupied  b) Unoccupied  c) no  d) finite

3) In an Linear Programming Problem functions to be maximized or minimized are called …………………

a) Constraints  b) objective function

c) basic solution  d) feasible solution

4) The assignment algorithm was developed by ………………….. method.

a) HUNGARIAN  b) VOGLES  c) MODI  d) TRAVELING SALESMAN

5) The assignment problem is a particular case of …………….

a) Transportation problem  b) Assignment problem

c) Travelling salesman problem  d) Replacement problem

6) Maximization assignment problem is transformed into a minimization problem by …………………

a) adding each entry in a column from the maximum value in that column

b) subtracting each entry in a column from the maximum value in that column

c) subtracting each entry in the table from the maximum value in that table

d) adding each entry in the table from the maximum value in that table

7) When the total demand is equal to the total supply then the transportation problem is said to be …………….

a) unbalanced  b) balanced  c) maximization  d) minimization

8) For finding an optimum solution in transportation problem ……………. method is used.

a) Simplex  b) Big – M  c) Modi  d) Hungarian

9) Linear Programming problem that can be solved by graphical method has ………..

a) quadratic constraints b) nonlinear constraints

c) bi- quadratic constraints d) linear constraints

10) Graphical method of linear programming is useful when the number of decision variables are …………

a) 1 b) 4 c) 2 d) 3

11) The area bounded by all the given constraints is called ……….

a) basic solution b) non feasible region

c) optimum basic feasible solution d) feasible region

12) For a two person zero – sum game, the value of game can be …

a) determined only if the pay-off matrix has a saddle point.

b) positive, negative, zero

c) determine only if the game is fair.

d) none of the above.

13) A game is said to be fair, if …..

a) upper value is more than lower value of the game.

b) upper value and lower value of the game is not equal.

c) upper value and lower value of the game are same and zero.

d) none of the above.

14) When maximin and minimax values of the game are same, then……

a) there is a saddle point. b) solution does not exist.

c) strategies are mixed. d) none of the above.

15) A mixed strategy game can be solved by

a) matrix method b) algebraic method

c) graphical method d) all of the above

16) The size of the pay-off matrix of a game can be reduced by using the principal of ……

a) dominance b) rotation reduction

c) game inversion d) game transpose

17) If the players select the same strategy each time, then it is referred to as …..

a) Mixed strategy b) Pure strategy

c) optimum strategy d) none of these

18) The dummy source or destination in a T.P. is introduced to

a) prevent solution to become degenerate.

b) to satisfy rim conditions.

c) ensure that total cost does not exceed a limit.

d) solve the balanced transportation problem.

19) If there is no saddle point then value of the game

a) lies between maximin and minimax

b) is equal to maximin

c) is equal to minimax

d) none of the above

20) If there are more than one saddle points then ...

a) only one solution b) there are more than one solution

c) no solution d) none of these

21) For unoccupied cells, unit cost difference $d_{ij} > 0$ then

a) it has no solution

b) it has an alternate optimal solution

c) we obtain basic feasible solution of transportation problem is solution d) none of these

22) A minimum element among the column maxima is called…...

a) maximin b) maximum c) minimax d) none of these

23) To resolve degeneracy at the initial solution, a very small quantity is allocated in ………... cell.

a) occupied b) no c) finite d) unoccupied

24) The assignment problem is always a ……. matrix.

a) circle b) rectangle c) triangle d) square

25) When the total demand is not equal to the total supply then the transportation problem is said to be …………….

a) unbalanced b) balanced c) maximization d) minimization

**Q. Long and short questions.**

1) The manager of an oil refinery must decide on the optimum mix of two possible blending processes of which the input and output production runs are as follows:

| Process | Input | | Output | |
|---|---|---|---|---|
| | Crude A | Crude B | Gasoline X | Gasoline Y |
| 1 | 6 | 4 | 6 | 9 |
| 2 | 5 | 6 | 5 | 5 |

The maximum amounts available of crudes A and B are 250 units and 200 units respectively. Market demand shows that at least 150 units of gasoline X and 130 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs. 4 and Rs. 5 resp. Formulate the problem for maximizing the profit.

2) Solve the problem by using Least-cost method

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Capacity |
|---|---|---|---|---|---|
| $O_1$ | 1 | 2 | 3 | 4 | 6 |
| $O_2$ | 4 | 3 | 2 | 0 | 8 |
| $O_3$ | 0 | 2 | 2 | 1 | 10 |
| Demand | 4 | 6 | 8 | 6 | |

3) Solve the problem by using Modi Method

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 3 | 7 | 6 | 4 | 5 |
| $S_2$ | 2 | 4 | 3 | 2 | 2 |
| $S_3$ | 4 | 3 | 8 | 5 | 3 |
| Demand | 3 | 3 | 2 | 2 | |

4) Solve the maximization assignment problem

| | P | Q | R | S |
|---|---|---|---|---|
| A | 2 | 3 | 4 | 5 |
| B | 4 | 5 | 6 | 7 |
| C | 7 | 8 | 9 | 8 |
| D | 3 | 5 | 8 | 4 |

5) Solve the following 2 X 2 game by using algebraic method

| | Player B | | |
|---|---|---|---|
| Player A | | $B_1$ | $B_2$ |
| | $A_1$ | 8 | -3 |
| | $A_2$ | -3 | 1 |

6) Solve the following travelling sales man problem

| | To city | | | | | |
|---|---|---|---|---|---|---|
| From city | | A | B | C | D | E |
| | A | ∞ | 2 | 5 | 7 | 1 |
| | B | 6 | ∞ | 2 | 8 | 2 |
| | C | 8 | 7 | ∞ | 4 | 7 |
| | D | 12 | 4 | 6 | ∞ | 5 |
| | E | 1 | 3 | 2 | 8 | ∞ |

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# B.Sc. III Paper DSE- E12 (Integral Transforms)

# Question Bank

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## Q. Multiple Choice Questions.

1) Laplace Transform of the function F(t) is

a) L {F (t)} = $\int_0^\infty e^{-pt}$ F(t) dt b) L {F (t)} = $\int_0^\infty e^{pt}$ F(t) dt

c) L {F (t)} = $\int_{-\infty}^\infty e^{-pt}$ F(t) dt d) L {F (t)} = $\int_{-\infty}^\infty e^{pt}$ F(t) dt

2) If F(t) is a function which is piecewise continuous on every finite interval in the range $t \geq 0$ and satisfies $| F(t) | \leq M e^{at}$ for all $t \geq 0$ and for some constants a and M, then

a) the Laplace transform of F(t) does not exists for all $p > a$

b) the Laplace transform of F(t) exists for all $p \leq a$

c) the Laplace transform of F(t) exists for all $p > a$

d) none of the above

3) The Laplace transform of function F(t) = sin at is………… $p > 0$

a) $p/ (p^2 + a^2)$ b) $p/ (p^2 - a^2)$ c) $a/ (p^2 + a^2)$ d) $a/ (p^2 - a^2)$

4) If L {F(t)} = f (p), then

a) L {F(at)} =a f ($\frac{p}{a}$) b) L {F(at)} = f ($\frac{p}{a}$)

c) L {F(at)} = $\frac{1}{a}$ f ($\frac{a}{p}$ ) d) L {F(at)} = $\frac{1}{a}$ f ($\frac{p}{a}$)

5) L {sin t cos t} =…………..., $p > 0$

a) $1/ (p + 2)$ b) $1/ (p^2 + 4)$ c) $p^2 + 4$ d) $p + 2$

6) Let F (t) be a continuous for all $t \geq 0$ and be of exponential order a as $t \to \infty$ and if $F^{I}$ (t) is of class A, then Laplace transform of the derivative $F^{I}$ (t) exists when…..

a) $p \leq a$ b) $p > a$ c) $p < a$ d) $p \geq a$

7) If F (t) is piecewise continuous and satisfies $| F (t) | \leq M e^{at}$ for all $t \geq 0$ for some constants a and M, then L {$\int_0^t F(x)\, dx$} = ………………. $p > 0$, $p > a$

a) $\frac{1}{p}$ L {F (t)} b) p L {F (t)} c) p d) $\frac{1}{p}$

8) If function F (t) is said to be of exponential order α as $t \to \infty$ if there exists a positive constant M, a number α and a finite number $t_0$ such that

a) $|e^{\alpha t} F(t)| < M$, for all $t \geq t_0$

a) $|e^{\alpha t} F(t)| > M$, for all $t \geq t_0$

a) $|e^{-\alpha t} F(t)| > M$, for all $t \geq t_0$

a) $|e^{-\alpha t} F(t)| < M$, for all $t \geq t_0$

9) Laplace transform of function $F(t) = t^n$ is ………., $p > 0$

a) $n! / (p^{n+1})$ b) $1 / (p^{n+1})$

c) $n / (p^{n+1})$ d) $(n + 1)! / (p^{n+1})$

10) If $L\{F(t)\} = f(p)$ and $G(t) = \begin{cases} F(t-a), & t > a \\ 0, & t < a \end{cases}$ then

a) $L\{G(t)\} = f(p)$ b) $L\{G(t)\} = e^{ap} f(p)$

c) $L\{G(t)\} = e^{-ap} f(p)$ d) none of these

11) If $f(p)$ is Laplace transform of $F(t)$, $p > \alpha$ then $f(p - a)$ is the Laplace transform of ………., $p > \alpha + a$

a) $e^{-at} F(t)$ b) $e^{at} F(t)$ c) $e^{\alpha t} F(t)$ d) $e^{-\alpha t} F(t)$

12) $L\{t^3 e^{-3t}\} =$

a) $3/(p + 3)^4$ b) $6/(p + 3)$ c) $3/(p + 3)$ d) $6/(p + 3)^4$

13) If $F(t)$ is a function of class of A and if $L\{F(t)\} = f(p)$ then

$L\{t^n F(t)\} =$……… where $n = 1, 2, 3, \ldots$

a) $(-1)^n \frac{d^n}{dp^n} f(p)$ b) $\frac{d^n}{dp^n} f(p)$

c) $(-1) \frac{d^n}{dp^n} f(p)$ d) none of these

14) If $L\{F(t)\} = f(p)$ then $L\{\frac{1}{t} F(t)\} =$…………

a) $\int_a^\infty f(p)\, dp$ b) $\int_p^\infty f(p)\, dp$ c) $-\int_a^\infty f(p)\, dp$ d) $-\int_p^\infty f(p)\, dp$

15) Laplace transform of function $F(t) = \cosh at$ is …...

a) $p / p^2 + a^2$ b) $a / p^2 - a^2$

c) $p / p^2 - a^2$ d) none of these

16) The sine integral is defined by

$S_i(t) =$ …………...

a) $\int_0^t \frac{\sin u}{u}\, du$ b) $\int_t^\infty \frac{\sin u}{u}\, du$

c) $\int_{-t}^t \frac{\sin u}{u}\, du$ d) none of these

17) The error function is defined by

$\operatorname{erf}(t) =$ ……….

a) $\frac{2}{\sqrt{\pi}} \int_0^\infty e^{-u^2}\,du$ b) $\frac{2}{\sqrt{\pi}} \int_0^\infty e^{-u}\,du$

c) $2 \int_0^\infty e^{-u^2}\,du$ d) none of these

18) L $\{\frac{1}{\sqrt{\pi t}}\}$ = …...

a) $\frac{2}{\sqrt{p}}$ b) $\frac{2}{\sqrt{2p}}$ c) $\frac{1}{\sqrt{p}}$ d) $\frac{1}{p}$

19) Laplace transform of $e^{at}$ is …

a) $\frac{1}{(p-a)}$ b) $\frac{1}{(a-p)}$ c) $\frac{2}{(p-a)}$ d) $\frac{2}{(a-p)}$

20) Heaviside's Unit function is defined by …

a) $H(t-a) = \begin{cases} 0, t \geq a \\ 1, t < a \end{cases}$ b) $H(t-a) = \begin{cases} 0, t < a \\ 1, t \geq a \end{cases}$

c) $H(t-a) = \begin{cases} a, t < a \\ 1, t \geq a \end{cases}$ d) $H(t-a) = \begin{cases} 0, t < a \\ a, t \geq a \end{cases}$

21) Linearity property, $a_1 F_1(t) + a_2 F_2(t)$ =…………

a) L $\{F_1(t)\}$ + L $\{F_2(t)\}$ b) $-a_1$ L $\{F_1(t)\}$ + $a_2$ L $\{F_2(t)\}$

c) $a_1$ L $\{F_1(t)\}$ - $a_2$ L $\{F_2(t)\}$ d) $a_1$ L $\{F_1(t)\}$ + $a_2$ L $\{F_2(t)\}$

22) If $L^{-1}\{f(p)\} = F(t)$, then $L^{-1}\{f(ap)\} = \frac{1}{a} F(\frac{t}{a})$ is

a) Second shifting theorem b) First shifting theorem

c) Linearity property d) Change of scale property

23) Laplace transform of $t^a$ is … where $a > -1$

a) $a!/p^{a+1}$ b) $(a+1)!/p^{a+1}$ c) $a!/p^a$ d) $a!/p^{a-1}$

24) $L^{-1}\{\frac{1}{p^2-a^2}\}$ = …….

a) sin hat b) $\frac{1}{a}$ sin at c) $\frac{1}{a}$ sin hat d) sin at

25) $\lim_{t\to 0} F(t) = \lim_{p\to\infty} p\, L\{F(t)\}$ is….

a) Final – value theorem b) Initial- value theorem

c) Division theorem d) Integral theorem

## Q. Long and short questions.

1) Define Laplace transform and show that Laplace transform of the function F (t) = $t^n$, $-1 < n < 0$, exist although it is not a function of class A.

2) Find L $\{\sin \sqrt{t}\}$.

3) Show that $L^{-1}\{\frac{s^2}{s^4+4a^4}\} = \frac{1}{2a}$ (cosh at sin at + sinh at cos at)

4) Find the Fourier sine and cosine transforms of the function $x^{m-1}$.

5) Find the Fourier transform of f(x) if f(x) = $\frac{\sin ax}{x}$

6) State and prove convolution theorem for Fourier transform.

7) Find the finite Fourier sine and cosine transforms of f(x) = x.

8) State and prove that Change of Scale property of inverse Laplace transform.

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# B.Sc. III Paper XIII (METRIC SPACE)

# Question Bank

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## Q.1. Multiple Choice Questions.

1)The metric space $\varrho(x,y) = \lceil x - y \rceil$ for x,y ∈R is called ……..metric.

a)absolute value b)discrete c)positive d)real

2)If $\varrho(x,y) = 1 \quad if\ x \neq y$

$= 0 \quad if\ x = y$ is a metric space.

This metric space is called………metric.

a)absolute value b)discrete c)positive d)real

3)In a metric space, any open ball is a……

a) continuous set b)convergent set c)closed set d)open set.

4) Finite intersection of open set is……….

a) continuous set b)convergent set c)closed set d)open set.

5)A metric space M is connected iff every continuous characteristic function is………..

a)continuous b)connected c)constant d)all a, b,c.

6) If every Cauchy sequence in metric space (M,$\varrho$) converges to a point in M then M is called ………..metric space.

a)convergent b) connected c)both a&b d) complete.

7)A compact subset of compact metric space is ……….

a)open b)closed c) both a&b d)neither a nor b

8) If the real valued function f is continuous on the closed bounded interval [a,b] then f is ………..on [a,b].

a)uniformly continuous b)uniformly discontinuous c) continuous d)none.

9) The union of infinite number of closed sets in a metrics space is …….set.

a)open b)closed c) both a&b d)neither a nor b

10) The metric space (0,1) with absolute value metric is ……..

a) totally bounded but not complete.

b) complete but not totally bounded.

c) both totally bounded and complete.

d)neither totally bounded nor complete.

11) The set of real numbers with absolute value metric is a metric space which is usually denoted by ………

a)$R^1$ b) $R^2$ c) $R_d$ d) $R^-$

12) Any subsets of $R_d$ is …………

a) closed b) open c) closed and open d)none of these

13) If A is bounded subsets of a metric space (M, $\rho$) then diameter of A is defined as diamA= ………

a) $\rho$(x,y), $\forall x, y \in A$ b) glb{ $\rho$(x,y)/$x, y \in A$ }

c) lub{ $\rho$(x,y)/$x, y \in A$ } d) $\infty$

14) If f is a continuous real valued function on the compact connected meteic space M then f takes on …… values between its minimum

**Q. 2) short and long answer.**

1) if the function the function $\rho$ defined by

$\rho$ (x, y ) = I x-y I

Then show that $\rho$ is metric for R.

2) If $l^2$ is the set of all sequences S = { Sn } then show that $Sn^2$ is convergent to $l^2$

3) If $\rho$ and $\sigma$ are both metrics for a metric for M then show that ($\rho + \sigma$) is also metric for M.

4) If ( M , $\rho$ ) is a metric space and a $\in$M f, g are real valued function whose domain are subsets of M and $\log_{x\to a} f(x) = L$ $\log_{x\to a} g(x) = N$ then prove that

$\lim_{x\to a}(f(x) + g(x)) = L + N$

5) If ( M , $\rho$ ) is a metric space and a $\in$M f, g are real valued function whose domain are subsets of M

and $\log_{x\to a} f(x) = L$ $\log_{x\to a} g(x) = N$ then prove that $\lim_{x\to a}(f(x) - g(x)) = L - N$

6) If ( M , $\rho$ ) is a metric space and a $\in$M f, g are real valued function whose domain are subsets of M

and $\log_{x\to a} f(x) = L$ $\log_{x\to a} g(x) = N$ then prove that $\lim_{x\to a}(f(x).g(x)) = L.N$

7) If ( M , $\rho$ ) is a metric space and a ∈M f, g are real valued function whose domain are subsets of M

and $\log_{x\to a} f(x) = L$ $\log_{x\to a} g(x) = N$ then prove that $\lim_{x\to a}(f(x)/g(x))$ =L/N

8) Define the convergenc sequence. Prove tha every convergent sequence is a Cauchy sequence.

9) prove that if Cauchy sequence have convergent subsequence then prove that whole sequence is convergent .

10) prove that every sequence in Rd is convergent.

11) Define open ball prove that every open ball is open set .

12) Define Cauchy sequence..Show that every convergent sequence in metric space is Cauchy sequence

13)Prove that, A function f is continuous iff inverse image of open set is open.

14)Let f be a continuous function on compact metric space $M_1$ into $M_2$.Then show that f($M_1$) is compact.

16).Let $\varrho: R \times R \to [0, \infty)$ be a function defined as

$\varrho(x, y) = 1 \ \ if\ x \neq y$

$= 0 \ \ if\ x = y$ for x,y∈R . Then show that $\varrho$ is metric on R.

17).If f is continuous then show that $|f|$ is also continuous.

18) Prove that in a metric space any open ball is open set.

19).Show that arbitrary union of open set is open.

20).Prove that, if A is totally bounded subset of M then A is bounded set.

21).Show that if M is compact metric space then M has Heine-Borel property.

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# B.Sc. III Paper XIV (Linear Algebra)

# Question Bank

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## Q.1. Multiple Choice Questions.

1) In vector space, Element of set V are known as…..
   a)scalar b)vector c)element d)none.
2) A non-empty subset W of a vector space V(F) is a subspace of V iff…..
   a)$\alpha x + \beta y \in W$ b) $\alpha x + \beta y \neq W$ $c)$ $\alpha x + \beta y = W$ d)all of the above
3) Sum of two subspace is ……..
   a)not subspace b)subspace c)vector space d)none
4) L(S) is ………..subset of subspace of V containing S.
   a)largest b)linear c)non-linear d)smallest
5) A non zero vector is always …………
   a)linearly independent b) linearly dependent
   c)not linearly independent d)not linearly dependent
6) A vector space ……… basis.
   a)only one basis b)only unit basis c)more than one basis d)none
7) If S & T are linear transformations and if ST is defines then …..
   a) always ST=TS b) need not be ST=TS c) always ST≠TS d) none
8) If T:V→W and S:W→U be two linear transformations then if ST is one-one then…..
   a)T is onto b)S is onto c)T is one-one d)S is one-one.
9) If V &W are vector space over field F and dim(V)=m, dim(W)=n then dim Hom(V,W)=……..
   a)mn b)m-n c)m+n d)m/n
10) The linearly independent set of vectors does not necessarily implies its ………
   a)Orthogonality b)Singularity c)inverse d)none
11) Inner Product space over real field is called -----
   a)null space b)subspace c)eclidean space d)unitary space
12) If T is a linear operator on $R^2$ defined by T(x,y)=(x-y,y),then $T^2(x,y)$=-----
   a)$(x^2,y^2)$ b) ( 2x-y,2y) c)(x-2y,y) d)(x,x-y)
13) Let λ=4 is an eigen value of an invertible operator T,then eigen value $T^{-1}$ ---
   a) 4 b)1/4 c)-4 d)-1/4
14) The norm of the vector (1,-3,2,-1) in the inner product space $R^4$ is ----
   a) $\sqrt{14}$ b)15 c) $\sqrt{15}$ d)4
15) The norm of vector u=(1,-2,3) is -----
   a) $\sqrt{30}$ b) $\sqrt{22}$ c) $2\sqrt{5}$ d) $6\sqrt{5}$

16) If A= $\begin{matrix} 0 & i \\ i & 0 \end{matrix}$ then characteristics polynomial of A is----

a)$x^2$+1 b)x c)x+1 d)1

17) If T:V→W is a L.T. and if dimV=7,dim rangeT=4 then dim kernel=

a)12 b)8 c)3 d)4

18) Let T:U→V be homomorphism and one-one then kernel of is ---

a)0 b)1 c){1} d){0}

19) If S is an orthonormal set then for α∊ S,------

a)α=1 b)α=0 c)$\|\propto\|$ =1 d) $\|\propto\|$ =0

20) If T(x,y,z)=(x-y,y-z,z-x) then kerT=------

a){(1,1,1)} b)(1,2,3) c){(1,2,4)} d)N.O.T

21) If A= $\begin{matrix} 3 & 0 \\ 0 & -2 \end{matrix}$ then the characterstics polynomial of A is---

i)$x^2$-6 ii) $x^2$+6 iii) $x^2$-x-6 iv) $x^2$+5x-6

22) If u=(4,-3,0,1) then norm of u with respect to Euclidean inner product is---

i) 30 ii) 26 iii) $\sqrt{30}$ iv) $\sqrt{26}$

23) The norm of vector (4,2,2,-6) is----

i)60 ii)14 iii)$2\sqrt{15}$ iv) $4\sqrt{15}$

24) Hom(V,W),where V and W are vector spaces over field F is called dual space of V over F,if------

i)V-F ii)W-F iii)W≠F iv)none of these

25) If T:$R^2$→$R^2$ and S:$R^2$→$R^3$ defined by T(x,y)=(y,x) and S(x,y)=(x+y,x-y,y)then ST(x,y)=-----

i) (y+x,y-x,x) ii) (x-y,x+y,x) iii) (x-y,x+y,y) iv) (y+2x,y-x,x)

**Que: Short and long Questions**

1. If T:V→V is a linear transformation ,then show that following statements are equivalent.

i)Range T∩ KerT={0} ii)if T(T(v))=0 then T(v)=0,v∊V

2)Prove that a non-empty subset W of vector space V(F) to be a subspace iff following conditions are satisfied : i)if x,y$\in W$ $then\ x+y \in W$

ii)if$x \in W$ & $a\ is\ scalar, \alpha \in F\ then\ \alpha x \in W.$

3) Prove that a non-empty subset W of a vector space V(F) is a subspace of V iff

$\alpha x+\beta y \in W;\ \ \alpha,\beta \in F, x, y \in W$

4) Obtain the eigen values, eigen vector and eigen space of matrix

A=$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

5) Let S ={( 1,4),(0,3)} be a subset of $R^2$(R) then show that (2,3)∈L(S).

6) If c≠ $o$ is an eigen values of an invertible operator T,then show that $c^{-1}$ is an eigen value of $T^{-1}$.

7) Show that vector (0,1,-2),(1,-1,0) and (1,2,1) are linearly dependent in $R^3$(R).
8) Suppose S be a finite subset of vector space V such that V =L(S) then there exist a subset of S which is a basis of V.

9) If V is a finite dimension vector space and S ={$v_1,v_2,v_3,\dots,v_n$} is a linearly independent subset of V then show that it can be extended to form a basis of V.

10) Define T : $V_1 \rightarrow V_3$ is defined by T(x)=(x,2x,3x) for all $x \in V_1$ then prove that T is linear map.

11) Prove that a linear transformation T : V→V is one-one if T is onto.

12) If T is a linear operator on V and if Rank$T^2$=Rank T,then show that

Range T$\cap$ ker $T = \{0\}$

13) If mapping T : $V_2 \rightarrow V_2$ is a linear map defined by T(2,3)=(3,4) and T(1,0)=

(2,0) then find rule for T($x_1,x_2$).

14) Obtain an orthonormal basis w.r.t the standard inner product space for the

subspaces of $R^3$ generated by (1,0,3),(2,1,1).

15) State and prove Sylvester's Law.

16)Define a linear span.Prove that L(S) is the smallest subspace of V containing S

17) State and prove Cauchy Schwarz's inequality.

18) State and prove triangle inequality and parallelogram law.

19) Let S be an orthogonal set of non-zero vectors is an inner product space V then

show that S is a linearly independent set .

20) Define range of L.T. T:V→W.Prove that range(T) is subspaces of W.

21) If T is linear transformation on a finite dimensional vector space V over F then

prove that c$\epsilon F$ *is an eigen* value of T iff T-cI is not invertible

22)If S and T be two subspaces of a vector space V,then prove that S∩T is also

subspace of V.

23) Find the eigen values for the matrix

$$A=\begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}.$$

24) Obtain an orthonormal basis w.r.t the standard inner product space for the

subspaces of $R^3$ generated by (0,1,-2),(1,-1,1) and (1,2,1).

25) Detemine whether map T:$R^3 \rightarrow R^2$ defined by T($x_1,x_2,x_3$)=($x_1,x_1+x_2+x_3$) is linear or not.

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# B.Sc. III Paper XV (Complex Analysis)

# Question Bank

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## Q.1. Multiple Choice Questions.

1. A point z=a at which a function f(z) is not analytic is known as………..

   a)Regular Point b)Singular Point c)analytic Point d)none of the above.

2. A function $\emptyset(x, y)$ is said to be harmonic z function if x&y satisfy……..

   a)Cauchy-Reimann equation b)Exact differential equation

   c)Laplace equation d)Polar equation

3.A continuous arc without multiple point is called ……….

   a)Jorden arc b)rectifiable arc c) continuous arc d)none

4. A curve is defined by z(t)=x(t)+iy(t) where t∈ $[a, b]$ is called closed curve if……

   a)$z(a) \ll z(b))$ b)$z(a) = z(b))$ c)$z(a) \neq z(b)$ d)$z(a) \gg z(b)$

5.Cauchy integral formula for simply connected domain is……

a)$\frac{1}{2\pi i} \oint \frac{f(\theta)}{\theta - z} d\theta$ b)$\frac{1}{2\pi i} \oint \frac{f(\theta)}{\theta + z} d\theta$ c )$\frac{1}{2} \oint \frac{f(\theta)}{\theta + z} d\theta$ d)none

   where z is any complex no. and $\oint$ *is a integration over Jorden curve C*.

6.Let f(z) is analytic function if point a is such that f(a)=0 then a is called...

   a)singular point b)analytic point c)isolated point d)zero's of f(z).

7.AN isolated singular point z=a is said to be……….singularity if principal part of f(z) contain infinite no. of terms.

   a)Removable b)pole c)essential d)non-isolated

8.If z=a is a pole of function f(z) which is limit of sequence of all poles of f(z) then z=a is known as …….

   a)non-isolated essential singularity b) non-isolated removable singularity

   c ) non-isolated pole singularity d)none

9.If a complex function is analytic at all finite points of the complex plane C then it is said to be …………….

   a)entire b)analytic c)regular d)continuous

10.$e^{1/z}$ has an ……………singularity.

a)removable b)pole c)essential d)non-isolated.

11.All ………functions are meromorphic functions on whole complex plane.

a)real b)rational c) irrational d)integer

**Que.2 Solve the following question**

1.Show that every differential function is continuous but converse is not true.

2.Prove that necessary and sufficient condition for f(z) to be analytic.

(Cauchy-Riemann equation)

3.Find the polar form of Cauchy-Riemann equation.

4.Explain the method of construction of analytic function f(z)=u+iv.

5.For the following u show that u is harmonic and also construct analytic function

a)u=$e^x \cos\theta$

b)u=$\log(x^2+y^2)$

6.Prove that an analytic function with constant modulus is constant.

7.Find the integral $\int_0^{1+i}(x-y+ix^2)$d

(i)along straightline from z=0 to z=1+

(ii)along real axis from z=0 to z=1 and then alon a line parallel to the imaginary axis from z=1 to z=1+i.

8.Evaluate the integral $\int_0^{1+i} z^2$dz.

9. State and prove Cauchy's theorem for simply connected domain and also for multiply connected domain.

9.State and prove Cauchy Integral Formula for simply connected damain.

10.If c is closed contour around the origin then prove that

$\left(\frac{a^n}{n!}\right)^2=\frac{1}{2\pi i}\oint\frac{a^n e^{az}}{n!\,z^{n+1}}\,dz$ Hence,deduce that $\sum_{n=1}^{\infty}\left(\frac{a^n}{n!}\right)^2=\frac{1}{2\pi}\int_0^{2\pi}e^{2a\cos\theta}\,d\theta.$

11.Define-i)Removable Singularity ii)poles iii)Essential singularity.

12.What kind of singularity for the following function

i)f(z)=$\frac{1}{\sin z-\cos z}$ ii)f(z)=$\frac{3z-2}{(z+4)(z+1)(z-1)^2}$

13.Find the residue of corresponding poles of the function

i)f(z)=$(1+z)^{-1}(1+z^2)^{-1}e^{iz}$ ii)f(z)=$\cot z$

14.State and prove Cauchy's Residue theorem.

15.Show that $\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)} = \frac{2\pi}{\sqrt{3}}$

16.Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$

17.State Jorden's lemma.

18.Show that $\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$

19.Define Entire function, Meromorphic function.

20.State Mittag –Leffler theorem and Argument theorem.

21.State and prove Rouche's Theorem

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## Department of Mathematics

## Class: B.Sc.-III Discrete Mathematics (DSE-F12)

## QUESTION BANK

## <u>Q. Select the correct alternatives for each of the following.</u>

1) Which of the following is not a proposition

a) Is mathematical boring ? b) Man landed on the sun last year.

c) Diamond is harder than graphite. d) He finished his work and went away.

2) If truth value of $p \leftrightarrow q$ is true, then truth value of $(p \wedge q) \vee (q \wedge p)$ is…..

a) T or F b) T c) F d) neither T nor F

3) $(p \wedge \sim q) \wedge (\sim p \vee q)$ is ……

a) contradiction b) tautology c) contingency d) either a or b

4) The negation of statement $5 \times 3 = 15$ or $7+5=12$ is …..

a) $5 \times 3 = 15$ or $7+5=12$ b) $5 \times 3 = 15$ or $7+5 \neq 12$

c) $5 \times 3 \neq 15$ and $7+5 \neq 12$ d) $5 \times 3 \neq 15$ or $7+5 \neq 12$

5) All premises are true then the conclusion is also true that means…..

a) valid argument b) invalid argument c) argument d) none of these.

6) The contribution of loop towards degree of the corresponding vertex is ….

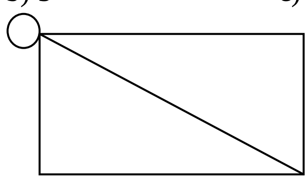
a) 0 b) 1 c) 2 d) $\infty$

7) A complete Km graph is …..

a) m-regular b) (m-1)-regular c) (m+1)-regular d) m(m-1)-regular

8) The Degree of first vertex is …..

a) 2 b) 5 c) 3 d) 4



9) A path containing all the edges in a graph is called ……

a) Hamiltonian path b) Eulerian path c) cycle d) Eulerian circuit

10) Let the incidence of matrix be given by $A = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then the number of loops in the multigraph is ……..

a) 1 b) 2 c) 0 d) 3

11) A binary tree is a rooted tree in which every parent has-----

a) atleast two children b)atmost two children c) two children d) exact two children

12) The argument form : p ∨ $q$ is called------

~p

∴ $q$

a) generalization b) specialization c) elimination d) transitivity

13) If truth value of $p \leftrightarrow q$ is true then truth value . (p ∧ $q$) ∨ (q∧ $p$) is---

a) T or F b) T c) F d) neither T nor F

14) The graph is connected iff ….

a) H is subgraph of G and H is connected.

b) No connected subgraph of G has H as a subgroup and contains vertieses or edges that are not in H.

c) both a and b

d) neither a nor b

15) Let u and v be distinct vertices of a tree T then there is …. From u to v in T.

a) one path b) precisely one path c) no path d) one cycle

16) In decimal number system, the number are represented with base ….

a) 2 b) 10 c) 8 d) 4

17) The graph contain no edge is called …..

a) empty graph b) regular graph c) simple graph d) complete graph

18) The maximum distance between a vertex to all other vertices is called ….

a) distance b) central c) eccentricity d) radius

19) Which simple graph have diameter one?

a) complete b) bipartite c) regular d) all of these

20) What is the truth value for statement variable p is true and q is false.

a) T b) T or F c) F d) none of these

# Q. Long Answer Type question.

1 ) Define: Graph, multi-graph and degree of vertex v of graph G. Prove that in a graph G, number of vertices of odd degree is even.

2) State and prove Hand- Shaking lemma. Prove that the maximum number of edges in graph consisting of n vertices is $\frac{n\,(n-1)}{2}$ .

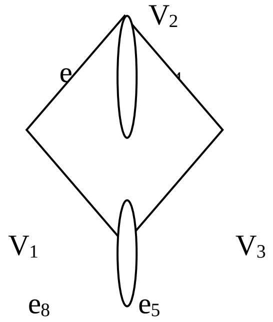
3) Define argument, premises and conclusion Use truth table to determine given argument is valid or invalid (p $\wedge$ $q$) $\longrightarrow \sim r$

$(p \vee \backsim q)$

$\sim q \rightarrow p$

$\therefore \sim r$

4) Define Eulerian path and Eulerian circuit. Determine the following graph have Euler circuit or not.




Prove that a graph has Euler circuit, then every vertex of the graph has positive even vertices.\

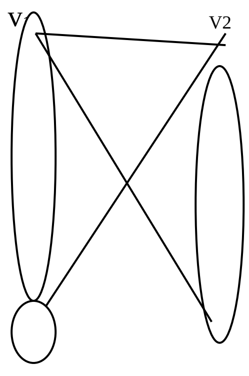
# Q. Short  Answer Type Question.

1) Give the converse, inverse and contrapositive of the following statement-

If differential function is continuous, then it is continuous.

2) Use De Morgan's laws to write negations for statement – Hal is a math measure and Hal's sister is complex science. Also use truth table and find the statement (p $\wedge$ $q$) $\vee$ (~p $\vee$( p $\wedge \backsim q$)) is tautology or contradiction.

3) Define adjacency matrix of a graph G. Find the adjacency matrix of the following graph G




Define incidence matrix with examples.

4) Determine whether the following argument argument form is valid or invalid by using truth table.

$(A \vee C) \rightarrow (H \rightarrow F)$

$(A \vee B) \rightarrow (F \rightarrow G)$

$A$

$\therefore H \rightarrow G$

5) Define logical equivalence. Show that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \longrightarrow r$ are logically equivalence to each other.

6) Prove that, G be graph with n edges, then G has Hamiltonian cycle if $m \geq \frac{1}{2}(n^2-3n+6)$ where n is number of vertices of G.