Course code	**:**	DSE – D6
Title of course	**:**	Integral Calculus
Theory	**:**	32 Hrs. (40 lecturers of 48 min.)
Marks	**:**	50 (Credit: 02)

Course Learning Outcomes: This course will enable the students to:

CO1: understand special functions.

CO 2: understand types of multiple integrals.

CO 3: apply special functions in applications.

CO 4: apply multiple integrals in real life problems.

Unit 1. Gamma and Beta Function. (16Hrs.)

1.1 Gamma function.

1.1.1 Definition of Gamma function and examples.

1.1.2 Properties of Gamma function.

1.1.2.1 $\Gamma(1) = 1$

1.1.2.2 $\Gamma(n+1) = n\,\Gamma(n)$ in general.

1.1.2.3 $\Gamma(n+1) = n!$ if n is positive integer.

1.1.2.4 $\Gamma(0) = \infty$, $\Gamma(\infty) = \infty$

1.1.2.5 $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$, $n > 0$

1.1.2.6 $\Gamma(n) = k^n \int_0^\infty e^{-kx} x^{n-1} dx$, $n, k > 0$

1.1.2.7 Examples based on article 1.1.2

1.2 Beta function.

1.2.1 Definition of Beta function and examples.

1.2.2 Properties of Beta function.

1.2.2.1 $\beta(m,n) = \beta(n,m);$ $m,n \geq 0$

1.2.2.2 $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta cos^{2n-1}\theta\, d\theta$; $m,n \geq 0$

1.2.2.3 $\int_0^{\frac{\pi}{2}} sin^p\theta\, cos^q\theta\, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$ p, q > -1

1.2.2.4 $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, m, n > 0

1.2.2.5 $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

1.2.2.6 $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$

1.2.2.7 $\beta(m,n) = a^n b^m \int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}} dx$

1.2.2.8 Duplication formula of Gamma function.

1.2.2.9 Examples based on article 1.2.2

Unit 2. Differentiation under integral sign, Error functions and Multiple integrals (16Hrs.)

2.1 Differentiation under integral sign

2.1.1 Leibnitz first rule of differentiation under integral sign.

2.1.2 Leibnitz second rule of differentiation under integral sign.

2.1.3 Examples based on articles 2.1.1 and 2.1.2

2.2 Error functions

2.2.1 Definition of $\mathrm{erf}(x)$, $\mathrm{erf_c}(x)$ and examples.

2.2.2 Properties of error functions.

2.2.2.1 $\mathrm{erf}(0) = 0,\ \mathrm{erf}(\infty) = 1$

2.2.2.2 $\mathrm{erf}(x) + \mathrm{erf_c}(x) = 1$

2.2.2.3 $\mathrm{erf}(-x) = -\mathrm{erf}(x)$

2.2.2.4 $\mathrm{erf_c}(-x) = 1 + \mathrm{erf}(x)$

2.2.2.5 $\mathrm{erf_c}(x) + \mathrm{erf_c}(-x) = 2$

2.2.2.6 Examples based on article 2.2.2

2.3 Multiple Integrals

2.3.1 Evaluation of double integrals in Cartesian form.

2.3.2 Evaluation of double integrals in Polar form.

2.3.3 Evaluation of double integrals in Cartesian form over the given region.

2.3.4 Evaluation of double integrals in Cartesian form by changing order of integration.

2.3.5 Evaluation of double integrals from Cartesian form to Polar form.

2.3.6 Proof of 1.2.2.4

Recommended Book:

1. P. N. Wartikar and J. N. Wartikar, A text book of Applied Mathematics, Pune Vidhyarthi Griha Prakashan, Pune. Vol. I, 2011.

Scope: Section III: Integral Calculus: Chapter XIV: 14.9,
Chapter XVI: 16.1 to 16.4, Chapter XIX: 19.1 to 19.3, Chapter XXI: 21.1 to 21.5

Reference Books:

1. Shantinarayan and Dr. P. K. Mittal, Integral Calculus, S. Chand and Company, New Delhi, 2020.
2. B. S. Grewal, Higher Engineering Mathematics, Khanna Publishers, Delhi, 2012.