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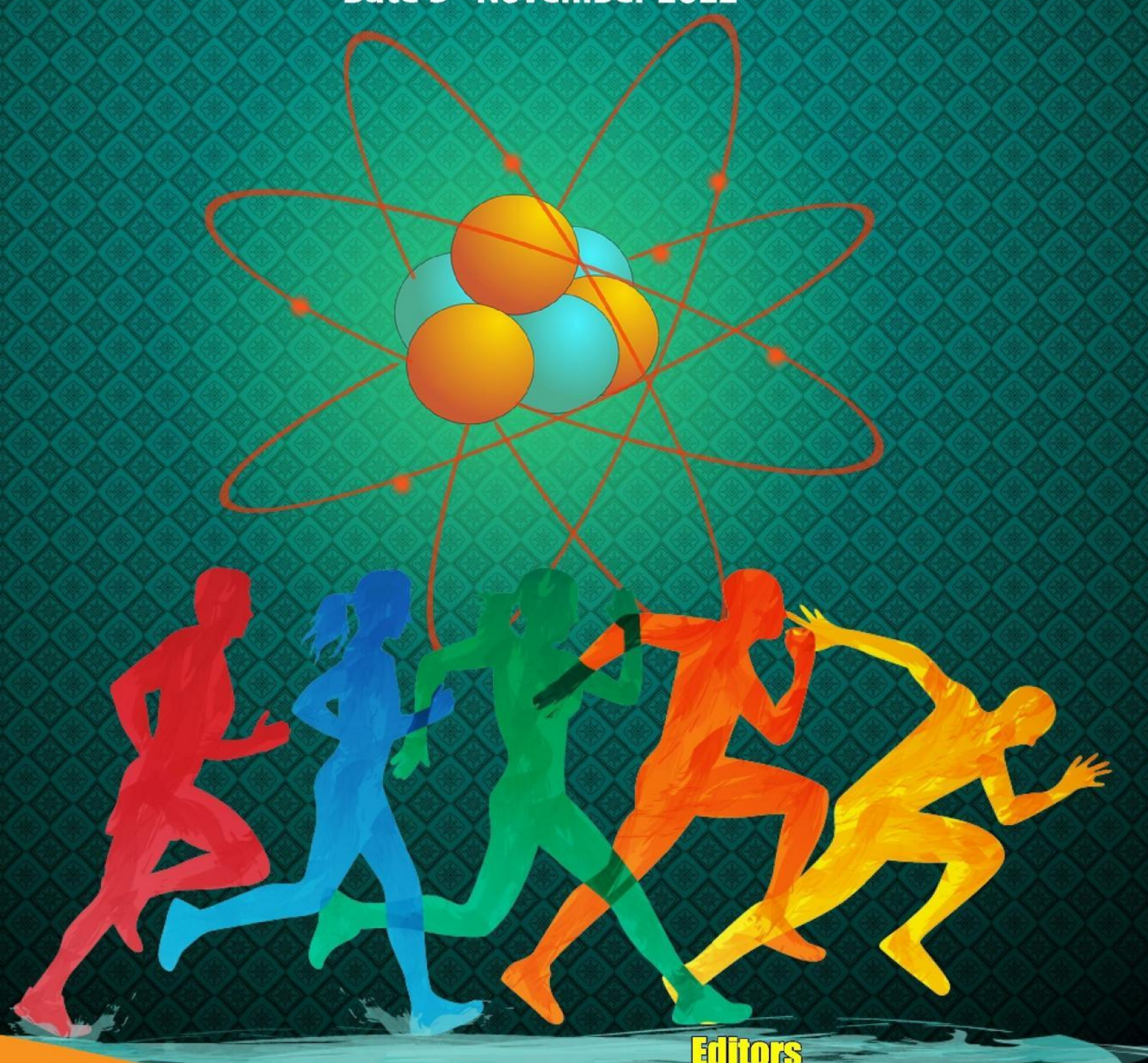

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Fuzzy Sets and Relations

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ABSTRACT:

Fuzzy mathematics is new concept in Mathematics.In Mathematics Fuzzy sets are sets whose elements have degrees of membership .Fuzzy sets are introduced by LotfiA.Zadeh and Dieter Klaua in 1965 as an extension of classical notation of Set .Fuzzy relations are used in different areas such as Linguistic ,decision making ,Clustering are special Cases of L-relations when Listhe unit interval [0,1].The present paper is to study concept about fuzzy relation the crisp relations these relations are different like reflexive, symmetric, transitive anti symmetric,I reflexive and order relations are considered in fuzzy.The applications of fuzzy relations are considered. The important contribution of the study will be recognition and relationship between known mathematical results the objectives are general i.e. to enrich the educational qualities in rural areas & to study the real life problem and using applications to obtain solution of it.

Keywords:-set, function, matrix, diagraph, co-ordinatesystem, cartesian-ordinates, domain, range.

Introduction:-

Fuzzy Set

In mathematics F**uzzysets** aresetswhose elements have degrees of membership.Fuzzy sets were introduced by Lotfi A. Zadeh and Dieter Klaua in 1965 as an extension of the classical notion of set. At the same time, Salii (1965) defined a more general kind of structures called *L*-relations, which were studied by him in an

abstract algebraic context. Fuzzy relations, which are used now in different areas, such as linguistics (De Cock, et al., 2000), decision-making (Kuzmin, 1982) and clustering (Bezdek, 1978), are special cases of *L*-relations when *L* is the unit interval [0, 1].

Objectives :

Product set

Let A and B be two nonempty sets, the product set or Cartesian product $A \quad B$ is defined as follows,

$$A \quad B = \{(a, b) \mid a \quad A, b \quad B\}$$

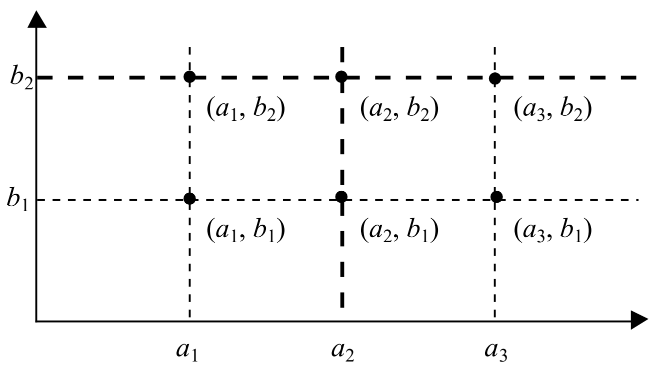
Extension to n sets

$$A_1 \quad A_2 \quad \ldots \quad A_n = \quad \{(a_1, \ldots, a_n) \mid a_1 \quad A_1, a_2 \quad A_2, \ldots, a_n \quad A_n\}$$

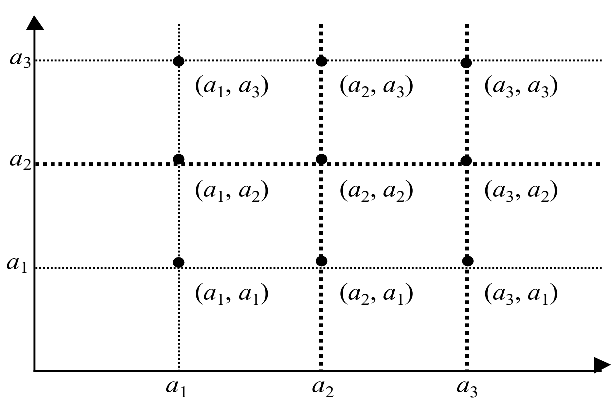
Crisp Relation

Example: $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2\}$

$$A \quad B = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2), (a_3, b_1), (a_3, b_2)\}$$



$$A \quad A = \{(a_1, a_1), (a_1, a_2), (a_1, a_3), (a_2, a_1), (a_2, a_2), (a_2, a_3), (a_3, a_1), (a_3, a_2), (a_3, a_3)\}$$



Cartesian product $A \quad A$

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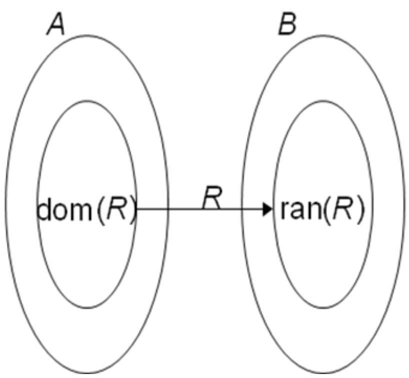
Binary Relation : $\boldsymbol{R = \{ (x,y) \mid x \quad A, y \quad B \} \quad A \times B \quad (x_1, x_2, x_3, \ldots , x_n) \quad R ,}$

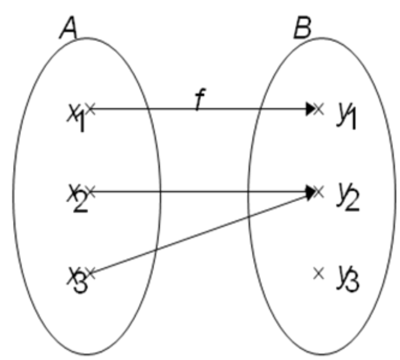
$\boldsymbol{R \quad A_1 \quad A_2 \quad A_3 \quad \ldots \quad A_n}$

Domain and Range

$dom(R) = \{ x \mid x \quad A, (x, y) \quad R$ for some $y \quad B \}$

$ran(R) = \{ y \mid y \quad B, (x, y) \quad R$ for some $x \quad A \}$





dom(R) , ran(R)　　　　　　　　Mapping $y = f(x)$

Characteristics of relation

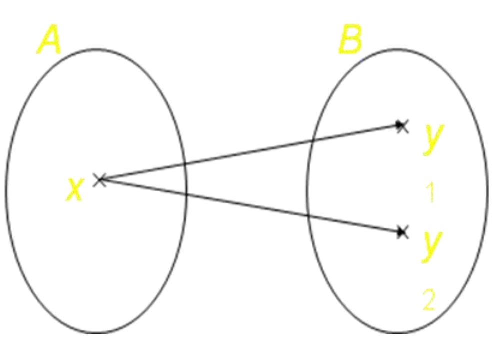
(1) One-to-many

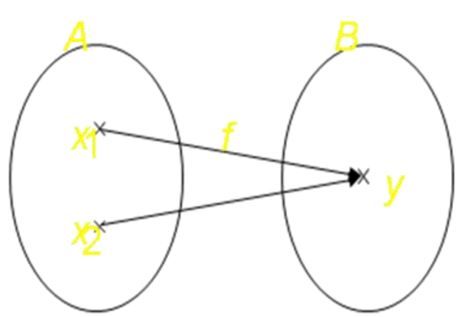
x　A, y1, y2　B (x, y1)　R, (x, y2)　R

(2) Surjection (many-to-one)

- $f(A) = B$ or $\text{ran}(R) = B$. $\quad y \quad B, \quad x \quad A, y = f(x)$

Thus, even if $x1 \quad x2$, $f(x1) = f(x2)$ can hold





One-to-many relation Surjection

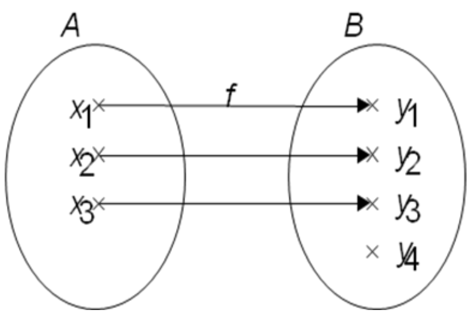
(*not a function*)

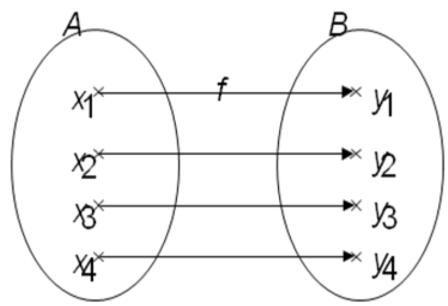
(3) Injection (into, one-to-one)

- for all x1, x2 A, x1 x2 , f(x1) f(x2).
- if R is an injection, (x1, y) R and (x2, y) R then x1 = x2.

(4) Bijection (one-to-one correspondence)

both a surjection and an injection





Injection Bijection

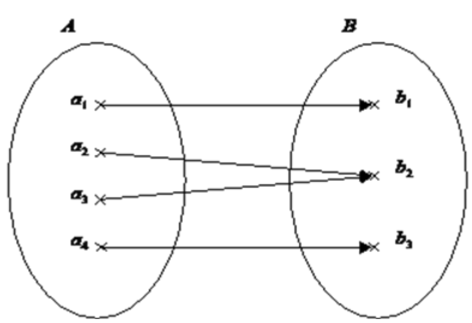
- Representation of Relations

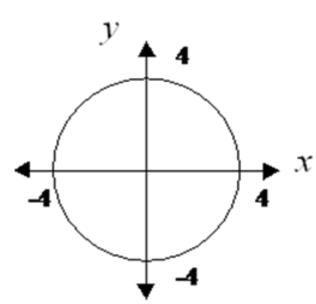
(1)Bipartigraph

representing the relation by drawing arcs or edges

(2)Coordinate diagram

plotting members of *A* on *x* axis and that of B on *y* axis





Binary relation from *A* to *B* Relation of $x^2 + y^2 = 4$

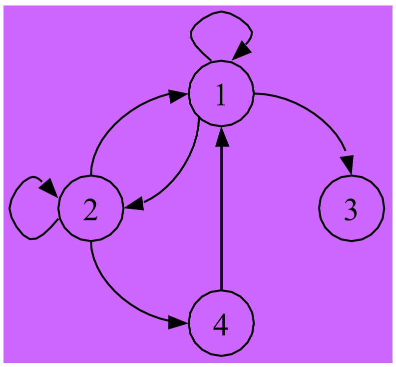
(3) Matrix

$$M_R = (m_{ij})$$

$$m_{ij} = \begin{cases} 1, & (a_i, b_j) \in R \\ 0, & (a_i, b_j) \notin R \end{cases}$$

$$i = 1, 2, 3, \ldots, m$$
$$j = 1, 2, 3, \ldots, n$$

(4) Digraph



Directed graph

- Operations on relations *R, S* *A* *B*

(1) Union T = R S

If (*x*, *y*) *R* or (*x*, *y*) *S,* then (*x*, *y*) *T*

(2) Intersection T = R S

If (*x*, *y*) *R* and (*x*, *y*) *S*, then (*x*, *y*) *T*.

(3) Complement

If (*x*, *y*) *R,* then (*x*, *y*) R^C

(4) Inverse

R^{-1} = {(*y*, *x*) *B* *A* | (*x*, *y*) *R*, *x* *A*, *y* *B*}

(5) Composition *T*

R *A* *B*, *S* *B* *C* , *T* = *S* *R* *A* *C*

T = {(*x*, *z*) | *x* *A*, *y* *B*, *z* *C*, (*x*, *y*) *R*, (*y*, *z*) *S*}

Types of Relation on a set

Reflexive : $x \quad A \quad (x, x) \quad R$ or $_R(x, x) = 1, \quad x \quad A$

Ireflexive : if it is not satisfied for some $x \quad A$

Antireflexive : if it is not satisfied for all $x \quad A$

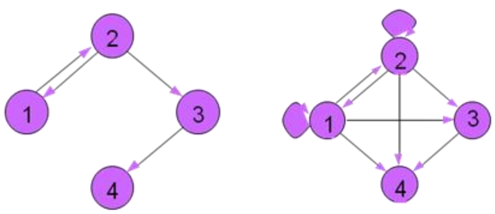
Symmetric : $(x, y) \quad R \quad (y, x) \quad R$ or $_R(x, y) = \quad _R(y, x), \quad x, y \quad A$

Asymmetric : when for some $x, y \quad A$, $(x, y) \quad R$ and $(y, x) \quad R$.

Antisymmetric : If for all $x, y \quad A$, $(x, y) \quad R$ and $(y, x) \quad R$

Transitive : For all $x, y, z \quad A$

$(x, y) \quad R, (y, z) \quad R \quad (x, z) \quad R$



(a) R (b) R^{∞}

Transitive Closure

Equivalence relation : Reflexive : $x \quad A \quad (x, x) \quad R$

Symmetric: $(x, y) \quad R \quad (y, x) \quad R$

Transitive relation: $(x, y) \quad R, (y, z) \quad R \quad (x, z) \quad R$

Fuzzy relation : $_R : A \quad B \quad [0, 1]$

- $R = \{((x, y), \quad _R(x, y)) | \quad _R(x, y) \quad 0 \,,\ x \quad A,\ y \quad B\}$

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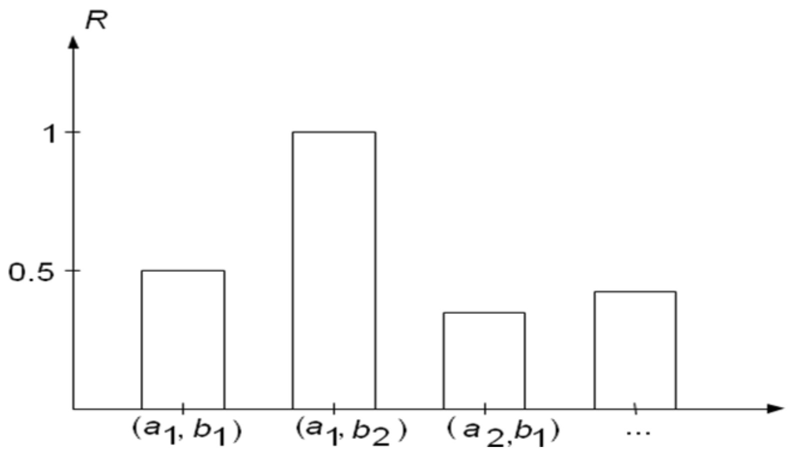
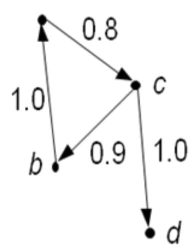


Fuzzy relation as a fuzzy set

- **Example**

Crisp relation R : $_R(a, c) = 1$, $_R(b, a) = 1$, $_R(c, b) = 1$ and $_R(c, d) = 1$.

Fuzzy relation R : $_R(a, c) = 0.8$, $_R(b, a) = 1.0$, $_R(c, b) = 0.9$, $_R(c, d) = 1.0$



A \ A	a	b	c	d
a	0.0	0.0	0.8	0.0
b	1.0	0.0	0.0	0.0
c	0.0	0.9	0.0	1.0
d	0.0	0.0	0.0	0.0

(a) Crisp relation (b) Fuzzy relation

Operation of Fuzzy Relation

1) Union relation : (x, y) A B

$_R$ $_S (x, y)$ = Max [$_R (x, y)$, $_S (x, y)$] = $_R (x, y)$ $_S (x, y)$

2) Intersection relation : $_R$ $_S (x)$ = *Min* [$_R (x, y)$, $_S (x, y)$] = $_R (x, y)$ $_S (x, y)$

3) Complement relation : (x, y) A B $_R (x, y) = 1 -$ $_R (x, y)$

4) Inverse relation : For all (x, y) A B, $_R{}^{-1} (y, x) =$ $_R (x, y)$

5) Standard Composition :For (x, y) A B, (y, z) B C,

$_{R\ \ S}(x, z) = \text{Max}\ [\text{Min}\ (\ \ _R(x, y),\ \ \ _S(y, z))]$

$\qquad y$

$= \quad [\ \ _R(x, y) \qquad _S(y, z)]$

$\qquad y$

$M_{R\ \ S} = M_R \quad M_S$

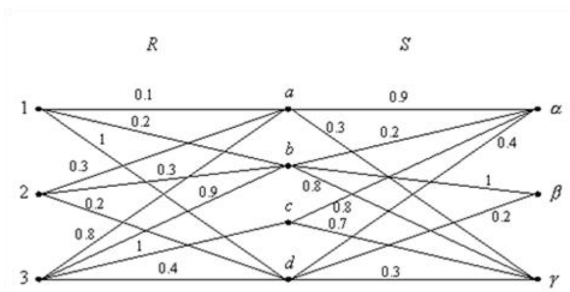
Example

S	α	β	γ
a	0.9	0.0	0.3
b	0.2	1.0	0.8
c	0.8	0.0	0.7
d	0.4	0.2	0.3

R	a	b	c	d
1	0.1	0.2	0.0	1.0
2	0.3	0.3	0.0	0.2
3	0.8	0.9	1.0	0.4

=>

S•R	α	β	γ
1	0.4	0.2	0.3
2	0.3	0.3	0.3
3	0.8	0.9	0.8



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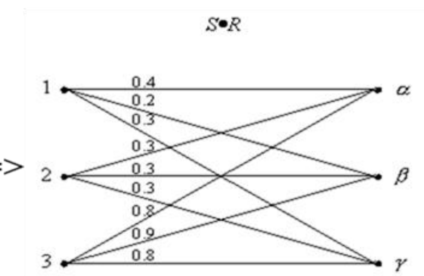

Composition of fuzzy relation

- Crisp: Transitive relation that contains R(X,X) with fewest possible members
- Fuzzy: Transitive relation that contains R(X,X) with smallest possible membership

Some applications of fuzzy Relation

1) Medical diagnosis 2)Medical expert systems 3)Information retrieval 4)Temperature dependence

5) Hybrid intelligent systems6) Diagnostic expert systems

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