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Department of Mathematics

Class: B.Sc.-II (Sem. III)

Real Analysis -I

QUESTION BANK 2022-2023

Q. Multiple choice Questions.

1) The set a+S = { a+s | s∈S } then sup (a+S) = …..
   a) inf (a+s) b) a + sup S c) a + sup (a+S) d) a + inf S

2) If $g(x) = x^2$ $(0 \leq x < \infty)$ then $g^{-1}(x) =$ ………
   a) x^2 b) x c) $x^{3/2}$ d) $x^{1/2}$

3) If f: A⟶B and If X ,Y are subset of A then………
   a) f(X∩Y) ≤ f(X) ∩ f(Y) b) f(X∩Y) = f(X) ∩ f(Y) c) f(X∩Y) ≥ f(X)∩f(Y) d) f(X∩Y) ≤ f(X)Uf(Y)

4) If f: A⟶B and if $f(a_1) = f(a_1)$ implies $a_1 = a_2$ for all a_1, a_2 ∈ A, then f is called ……. Function.

   a) onto b) one – one c) one – one and onto d) none of these

5) Any non-empty subset of real numbers which is bounded below has ……

   a) infimum b) both infimum and supremum

   c) supremum d) Neither infimum nor supremum

6) If f(x) = sinx then $f^{-1}(1)$ is ……….

   a) π/2 +2nπ b) π/2 -2nπ c) π/2 - nπ d) π/2 +2π

7) For each value of n∈N, $1^2+2^2+3^2+\ldots+n^2$ = ……….

   a)(1/3) n(n+1)(2n+1) b) [n(n+1)(2n+1)] / 6 c) [n(n+1)] /2 d) $[2n^2(n+1)] / 2$

8) In the following which is not example of Denumerable set.

   a) odd natural numbers b) integers c) irrational numbers d) rational numbers

9) Set of real numbers is …….

   a) uncountable b) countable c) finite d) none of these

10) If f is a function from A into B with the range f = B then f is called ……

    a) onto b) one – one c) one – one and onto d) none of these

11) A function f: A⟶B is called a one – one correspondence between A and B, if……..

    a) f is one-one but not onto b) f is one – one and onto

c) f is not one – one but onto d) f is neither one –one nor onto

12) The set of ……… is uncountable set.

a) positive integers b) integers c) rational numbers d) real numbers

13) If f(x) = 3x and g(x) = $2x^3$-4x+1 then (f°g)(x) = …….

a) $2x^4$-4x+5 b) $6x^3$-12x+1 c) $6x^3$+12x+1 d) $3x^3$+12x+1

14) Let f be a real valued functions described by f(x) = x^2 ($-\infty < x < \infty$). Then f([0,3]) = ……….

a) (0,9) b) (0,9] c) [0,9) d) [0,9]

15) The greatest lower bound of set of all positive even integers integers is……

a) 0 b) 2 c) 1 d) none of these

16) The least upper bound of the set { 1/n| n∊N } is ……

a) 1 b) 0 c) -1 d) none of these

17) Between any two distinct real numbers there exist ……

a) only one rational number b) finite numbers of rational numbers

c) infinitely many rational numbers d) None of these

18) If u is an Upper bound of a set A of real numbers and u∊A, then u is …….

a) infimum of A b) both infimum and supremum of A

c) supremun of A d) Neither infimum nor supremum

19) A subset A of real numbers is said to be bounded if it is ……

a) bounded above b) bounded above as well as bounded below

c) bounded below d) None of these

20) Supremum of the set { 1, 1/2, 1/3, 1/4, …..} is …….

a) 0 b) 1 c) ½ d) ¼

21) Every non-empty subset S of R which has a lower bound has …..

a) infimum b) supremum c) no infimum d) none of these

22) If x and y are real numbers, which one of the following is always true?

a) $|x - y| \leq |x| - |y|$ b) $|x - y| \geq |x| + |y|$ a) $|x - y| \geq ||x| - |y||$ a) $|x - y| = |x| - |y|$

23) Composite functions of two bijective function is …….

a) one-one b) onto c) bijective d) injective

24) If A, B, C are any sets then A∩ (B-C) = ……

a) (A∩B)-C b)(A- B)UC c) (A-C)∩ (B-C) d) (A∩B)- (A∩C)

25) If $x > -1$ then …..

a) $(1 + x)^n \geq 1 + nx$ b) $(1 + x)^n > 1 + nx$ c) $(1 + x)^n \leq 1 + nx$ a) $(1 + x)^n \geq 1 + 2nx$

Q. Long and Short answers type questions

1) Show that if $f : A \longrightarrow B$ and G, H are subsets of B, then $f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$ and

$f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$.

2) State and prove the Bernoulli's Inequality and find the all real numbers x that satisfy the inequality, $\frac{1}{x} < x^2$.

3) Prove that, $3 + 11 + \ldots + (8n - 5) = 4n^2 - n$ for all $n \in N$.

4) Prove that, any subset of S of R which contains (0, 1) is uncountable.

5) If $0 < a < b$, show that a) $a < \sqrt{ab} < b$ and b) $\frac{1}{b} < \frac{1}{a}$

6) Show that, if x, y are rational numbers then $x + y$ and xy are rational numbers.

7) Find all $x \in R$ that satisfy both $|2x - 3| < 5$ and $|x + 1| > 2$ simultaneously.

8) Prove that x and y are real numbers with $x < y$, then there exist an irrational number z such that

$x < z < y$.

9) Prove that a non-empty set T_1 is finite iff there is a bijection from T_1 onto a finite set T_2.

10) Prove that, if A and B are countable then $A \cap B$ is countable.

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Class: B.Sc.-II (Sem. III)

Algebra- I (DSC-6C)

QUESTION BANK 2022-2023

Q. Select the correct alternatives for each of the following.

1) If A and B are Hermitian matrices then A + B is ….

a)Hermitian b)Skew – Hermitian c)Symmetric d)Skew-symmetric

2) If the system of equations has one or more solutions then the equations are ….

a) inconsistent b) consistent c)homogeneous d)non-homogeneous

3) The sum of eigen values of a matrix is ….

a) equal to trace of matrix b) not equal to trace of the matrix

c) 0 d) none of these

4) A square matrix A = $[a_{ij}]$ is Hermitian if ….

a) a_{ij} = - a_{ji} for all i & j b) $a_{ij} = a_{ji}$ for all i & j

c) $a_{ij} = -\overline{a_{ji}}$ for all i & j d) $a_{ij} = \overline{a_{ji}}$ for all i & j

5) The rank of null matrix is ….

a) 0 b) 1 c) n d) ∞

6)If R is a relation on set A={1,2,3,4,5,6,7,8,9,10} defined as (x, y) ϵ R iff x is square root of y ,Then R is ….

a){(1,1),(4,16),(5,25)} b){(1,1),(1,2),(2,4)}

c) {(1,1),(3,9),(4,2)} d){(1,1),(2,4),(3,9)}.

7) A relation R on set A is transitive if and only if ….

a) $R \circ R \subseteq R$ b) $R \subseteq R \circ R$ c) $R^{-1} \subseteq R$ d) $R \subseteq R^{-1}$

8) The strict order relation on set of integers is ---

a) only reflexive but neither symmetric nor transitive

b) only transitive but neither reflexive nor symmetric

c) reflexive, asymmetric and transitive

d) irreflexive, symmetric and transitive

9) Let A = {a, b} and B = { x, y, z} then the number of element in A x B is….

a) 2 b) 3 c) 6 d) 5

10) A relation R on a set S which reflexive, symmetric and transitive is called …. Relation.

a) inverse b) partial order c) an equivalence d) anti-symmetric

11) If G ={1,-1,-i, i} is a group under multiplication operation then O(-i) = ….

a) 1 b) 0 c)2 d)4

12) Let G be a group and a ∈ G be arbitrary. If N (a) is the normalizes of a then for every x ∈ N (a), we have ….

a) xa = ax b) xa = e c) ax = e d) $xaa^{-1} = e$

13) If G is a group then for all a, b ∈ G ….

a) $(ab)^{-1} = a^{-1} b^{-1}$ b) $(ab)^{-1} = b^{-1} a^{-1}$ c) $(ab)^{-1}= (ba)^{-1}$ d) $(ab)^{-1}= ab$

14) If e_1 and e_2 are two identity elements in a group G then ….

a) e_1 and e_2 are distinct b) $e_1 = k\, e_2$ c) $e_1 = e_2$ d) $e_1 + e_2$

15) Let $\mathbb{Z}_5^+$ be the set of non- zero integers modulo 5. In ($\mathbb{Z}_5^+$, $\odot_5$), the multiplicative inverse of 2 is ….

a) 1 b) ½ c) 2 d) 3

16) If G is non-abelian group then….

a) G has no any left coset b) right coset and left coset need not be equal

c) right coset and left coset must be equal d) none of these

17) If a is a generator of a cyclic group G, then......

(a) O (a) ≠ O (G) (b) O (a) = O (G) (c) O (a) < O (G) (d) O (a) > O (G)

18) The order of the identity element in any group is ....

(a) two (b) zero (c) one d) none of these

19) Cyclic group may have ..... generator.

(a) one and only one (b) exactly two (c) more than one (d) none of these

20) The group { 2^n | n∈ Z} under multiplication operation is ...... group of complex numbers (C^*, ·)

(a) an infinite cyclic subgroup (b) a finite cyclic subgroup

(c) not cyclic subgroup (d) none of these

Q. Long answer type questions

1) Find the values of λ for which the system has non-trivial solution and solve it.

$2x-2y+z=\lambda x,\ 2x+3y+2z=\lambda y,\ -x+2y=\lambda z$

2) Show that the following equations are consistent and solve them. $3x+y+z=8,\ -x+y-2z=-5,\ 2x+2y+2z=12,\ -2x+2y-3z=-7$

3) Find the eigenvalues and eigenvectors of the following matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

4) Verify Cayley- Hamilton theorem for matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ and compute A^{-1} & A^4

5) Let A = {1,2,3,4} and B= {x,y,z} be two sets and R be a relation from A to B given by R= {(1,y), (1,z), (3,y), (4,x)}

i) Draw the coordinate diagram of R (ii) Draw the arrow diagram of R

iii) Determine the matrix of R iv) Write matrix of R

v) Draw the graph and Find R^{-1}

6) Define (i) Equivalence relation (ii) Partial order relation. Let L be set of all straight lines in a plane. Define a relation P on L by $(x,y) \in P$ iff x is parallel to y. Show that P is an equivalence relation.

7) If R = {(a, b), (b,c), (c, d), (b, a)} is a relation on set A={a,b,c,d} then find the transitive closure of R by using Warshal's algorithm.

8) Prove that, X be a non-empty set and let ~ an equivalence relation on set X.Let $x, y \in X$ then $y\in [x]$ iff $[x]= [y]$

9) Prove that, In any group G, (i) Identity element is unique. (ii) Inverse of each element $a\in G$ is unique (iii) $(a^{-1})^{-1} = a$, for all $a\in G$ (iv) $(ab)^{-1}= b^{-1}a^{-1}$, for all $a, b\in G$ (v) $ab = ac \Rightarrow b = c$ and $ba = ca \Rightarrow b = c$, for all $a, b, c \in G$

10) Define Center of group. Let G be a group of all 2x2 non-Singular matrices over $\mathbb{R}$ under matrix multiplication. Find the center of G.

11) Prove that, If H is a subgroup of G and $a,b \in G$, then

(i) $Ha= H$ if and only if $a \in H$.

(ii) $Ha= Hb$ if and only if $ab^{-1}\in H$

(iii) Ha is a subgroup of G if and only if a∈ H.

12) Prove that, If a is a generator of a cyclic group G, then O (a) = O (G). i.e., Order of a cyclic group is equal to the order of its generator.

Q. <u>Short answer type questions</u>

1) Show that A and B are Hermitian or Skew-Hermitian then A+B is Hermitian or Skew-Hermitian.

2) Show that the matrix A = $\begin{bmatrix} 3i & 7+4i & -2+5i \\ -7+4i & -2i & 3-i \\ 2+5i & -3-i & 0 \end{bmatrix}$ is Skew-Hermitian.

3) Find the eigen values of matrix B = $\begin{bmatrix} 2 & 4 & -6 \\ 4 & 2 & -6 \\ -6 & -6 & -15 \end{bmatrix}$

4) Let A = {a,b,c,d } be a set and R= {(b, b) (b, c) , (c, c), (c,d)} and S = {(b, a), (c, b), (d, c)} be two relations on set A then find R^{-1} and RUS.

5) Prove that, The inverse of on equivalence relation is also an equivalence relation.

6) Prove that A relation R on a set is symmetric iff R = R^{-1}.

7) Let A= {1, 2, 3} and R= {(1,1), (1,2), (1,3), (2,3), (3,1),(3,2)}. Find the transitive Closure of R.

8)Define cyclic group .Prove that subgroup of a cyclic group is cyclic.

2)Define subgroup of G.Prove that a non empty subset H of a group G is a subgroup of G iff $a,b\epsilon H$ implies that $ab^{-1}\epsilon H$.

3)Show that centre of a group G is subgroup of G.

4)Prove that order of a cyclic group is equal to order of its generator.

5)Prove that a cyclic group is abelian.

6) A={1,2,3,4},B={x,y,z} be sets R is a relation from A to B given by R={(1,y),(1,z),(3,y),(4,x)} subset A×B.
i)Draw the coordinate diagram of R.ii)Determine the matrix of R.
iii) Find domR and ranR iv)Draw the arrow diagram of R

7)Find the eigenvalues and eigenvectors $\begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

8)Solve x+3y-2z=0,x-y+2z=0,x-11y+14z=0.

9)If R^{-1} and S^{-1} are the inverse of the relations R and S respectively then $(R0S)^{-1}=S^{-1}0R^{-1}$.

10)Let A={a,b,c,d} be a set andR={(a,a),(a,c),(c,b),(c,d),(d,b)},S={(b,a),(c,c),(c,d),(d,a)} be two relations on set A.Verify that $(R0S)^{-1}=S^{-1}0R^{-1}$.

11)Define Symmetric,Antisymmetric,transitive reflexive and Partial order Ralation.

12)Prove that the inverse of an equivalence relation is also an equivalence relation.

13)Prove that a relation R on a set is symmetric iff $R=R^{-1}$.

14)Prove that let R be a relation on a set A .Then R^{∞} is the smallest transitive relation on A that contains R.

15)Define Quotient Set.Prove that let X be a nonempty set and let ~ be an equivalence relation on the set X.Let x,yϵX.Then yϵ[x] iff [x]=[y].

16)Show that the relation ~ is a equivalence relation in the set R of real numbers given by ~={(a,b)/a-bϵQ}.

17)Show that the following equations are consistent and solve then
3x+y+z=8 , -x+y-2z=-5,
2x+2y+2z=12, -2x+2y-3z=-7.

18)Prove that Every subgroup of a cyclic group is cyclic.

19)If H is a subgroup of G and a,bϵG,then H a=H iff aϵG ii)Ha is a subgroup of G iff aϵH.

20)Prove that Intersection of subgroups of a group G is a subgroup G.

Ans 1) a 2) b 3) a 4) d 5) a 6) d 7) a 8) b 9) 6 10) c 11) d 12) a 13)b 14) c 15) d
16) b 17) b 18) c 19) c 20) a

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Department of Mathematics

Class: B.Sc.-II

Real Analysis II (DSC-5D)

QUESTION BANK 2021-2022

Que. Choose correct alternative from the following.

1) A sequence$\{Sn\}_{n=1}^{\infty}$ is called null sequence if it …..

a) converges to 0 b) converges to 1 c) diverges to ∞ d) diverges to -∞

2) If C= {Cn}={$\sqrt{n}$} and N={n_i}={i^4}, i∈ N then C°N =……

a) {i} b) {i^2} c) {i^3} d) {i^4}

3) $\lim_{n\to\infty} (1+\frac{1}{n})^{\frac{n}{2}}$ =……

a) e b) $\frac{1}{e}$ c) $\sqrt{e}$ d) e^2

4) Non decreasing sequence which is bounded above is ……

a) divergent b) oscillatory c) convergent d) none of these

5) If $\{Sn\}_{n=1}^{\infty}$ is a sequence of non negative real numbers and $\lim_{n\to\infty} Sn$ = L then……

a) L=0 b) L=1 c) L≤ 0 d) L≥1

6) If {Sn}= $\{(-1)^n\}_{n=1}^{\infty}$ then $\lim_{n\to\infty} Sup\ Sn$= ……….

a) 0 b) 1 c) -1 d) 2

7) If {Sn}= $\{sin\frac{n\pi}{2}\}_{n=1}^{\infty}$ then $\lim_{n\to\infty} Sup\ Sn$ =……. and $\lim_{n\to\infty} Inf\ Sn$ =……

a) 1,1 b) 1,-1 c) -1,1 d) -1,-1

8) If {Sn}= {10^{100}, 1, -1, 1, -1, 1, -1,……} then $\lim_{n\to\infty} Inf\ Sn$ =……

a) 1 b) -1 c) 10 d) 10^{100}

9) {Sn}= { 1, 0, 1, 0, 1, 0, …..} is (C, 1) summable to ….

a) 1 b) $\frac{1}{2}$ c) 2 d) 0

10) A sequence {Sn}={n} is …….

a) (C, 1) summable to 0 b) (C, 1) summable to 1

c) (C, 1) summable to -1 d) not (C, 1) summable

11) A necessary condition for the convergence of the finite series $\sum u_n$ is ……

a) $\lim_{n\to\infty} u_n \neq 0$ b) $\lim_{n\to\infty} u_n = 0$ c) $\lim_{n\to 0} u_n = 0$ d) $\lim_{n\to 0} u_n \neq 0$

12) The positive p-series $\sum \frac{1}{n^p}$ is divergent for……

a) p<1 b) $p \geq 1$ c) $p \leq 1$ d) p>1

13) Consider the statement i) $\sum \frac{1}{n}$ is divergent, ii) $\sum \frac{1}{n^2}$ is convergent then……

a) Both i) and ii) are true b) Both i) and ii) are false

c) only i) is true d)only ii) is true

14) If $\sum u_n$ is series of positive terms with $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = L$ then the series diverges if……

a) L<1 b) L>1 c) L=0 d) None of these

15) For the convergence of series $\sum u_n$ the condition $\lim_{n\to\infty} u_n = 0$ is…….

a) only necessary b) Sufficient

c) Necessary and sufficient d)None of these

16) The alternating series $\sum \frac{(-1)^{n+1}}{n^p}$ is divergent for…….

a) p>1 b) $p \geq 1$ c) $0<p \leq 1$ d) 0<p<1

17) Which of the following is true?

a) Every convergent series absolutely convergent

b) Every absolutely convergent series is convergent

c) Every convergent series is conditionally convergent

d) Every conditionally convergent series is absolutely convergent

18) The series $\sum \frac{(-1)^{n+1}\log(n+1)}{(n+1)^2}$ is……. Series

a) Divergent b) Convergent

c) conditionally convergent d) None of these

19) The series $\sum \frac{x^n}{n!}$ converges absolutely for…….

a) All values of x b) $x \geq 0$ c)x>0 d) $0<x \leq 1$

20) The series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ is…….

` a) Convergent b) Divergent c) Cannot decided d) None of these

Que.2) Solve short answer questions

1) If S= $\{Sn\}_{n=1}^{\infty} = \{\frac{1}{n}\}_{n=1}^{\infty}$ and N={n_i} ={2^i} ; iϵN then find S°N

2) If $\{Sn\}_{n=1}^{\infty}$ is a sequence of real numbers if $S_n \leq M$ and if $\lim_{n\to\infty} S_n$= L then prove that L$\leq M$

3) Prove that $\lim_{n\to\infty} \frac{3n^2-6n}{5n^2+4} = \frac{3}{5}$

4)Prove that $\lim_{n\to\infty} (1+\frac{1}{2n})^{n+1}$ = e also prove that $\lim_{n\to\infty} (1+\frac{1}{n+1})^{n}$ = e

5) Prove that Every convergent sequence is bounded

6)Prove that if $\{Sn\}_{n=1}^{\infty}$ is a convergent sequence of real numbers then $\lim_{n\to\infty} Inf\ S_n = \lim_{n\to\infty} S_n$

7) If $\{Sn\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequence of real numbers and $S_n \leq t_n$ then

a) $\lim_{n\to\infty} Sup\ S_n \leq \lim_{n\to\infty} Sup\ t_n$

b) $\lim_{n\to\infty} Inf\ S_n \leq \lim_{n\to\infty} Inf\ t_n$

8) Define Cauchy sequence and prove that If the sequence of real numbers $\{Sn\}_{n=1}^{\infty}$ is a converges then $\{Sn\}_{n=1}^{\infty}$ is Cauchy sequence

9) Find $\lim_{n\to\infty} Sup$ if {S_n} = { 1, -1, 1, -2, 1, -3 }

10) Find $\lim_{n\to\infty} Inf$ if {S_n} = $\{(-1)^n\}_{n=1}^{\infty}$

11) If $\{Sn\}_{n=1}^{\infty}$ is a sequence of real numbers then prove that

$\lim_{n\to\infty} Inf\ S_n \leq \lim_{n\to\infty} Sup\ S_n$

12) Prove that the sequence { 1, 0, 0, 1, 0, 0, 1, 0, 0,….} is (C,1) summable

13) Verify (C,1) summability of sequence {S_n} = $\{n^2\}_{n=1}^{\infty}$

14) Find the following series are convergent or divergent

a) $\sum_{n\to 1}^{\infty} \frac{n+2}{10^{10}\,(n+4)}$ b) $\sum_{n\to 1}^{\infty} cos\frac{\pi}{n}$

15) Prove that the positive term geometric series$\sum_{r\to 0}^{\infty} r^n$ converges for r < 1 and Diverges to infinity for r ≥ 1

16) Test the convergence of series

a) $\sum u_n = \sum (\sqrt{n^4+1} - \sqrt{n^4-1})$

b) $\sum u_n = \frac{1}{\sqrt{1.2.3}} + \frac{1}{\sqrt{2.3.4}} + \frac{1}{\sqrt{3.4.5}}$

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Department of Mathematics

Class: B.Sc.-II

Algebra-II (DSC-6D)

QUESTION BANK 2022-2023

Que. Choose correct alternative from the following.

1) If H is a subgroup of a finite group G and o(H) = 4, o(G) = 36 then [G:H] = ……

a) 3 b)12 c) 4 d) 9

2) If G is a finite group and H is a subgroup of G then $i_G(H)$ = …..

a) $\frac{o(H)}{o(G)}$ b) o(G) c) o(H) d) $\frac{o(G)}{o(H)}$

3) The theorem that for any integer a and prime p, $a^p \equiv a \pmod p$ is called ….

a) Fermat's theorem b) Euler's theorem

c) Sylow's first theorem d) Lagrange's theorem

4) If H is a subgroup of a finite group G then ……

a) $\frac{[G:H]}{o(H)}$ b) $\frac{[G:H]}{o(G)}$ c) $\frac{o(G)}{[G:H]}$ d) $\frac{o(H)}{[G:H]}$

5) $\emptyset(21) =$ ……

a) 6 b) 12 c) 20 d) 14

6) A subgroup H of a group G is said to be a normal subgroup of G if for all g∈ G and for all h∈ H …..

a) $ghg^{-1} \in H$ b) Hg=gH c) both a) and b) d)Neither a) nor b)

7) Every subgroup of an abelian group is …….

a) normal subgroup b) abelian but not normal

c) neither abelian nor normal d) none of these

8) Let H be a subgroup and K is normal subgroup of the group G, then ….. is normal in H.

a) H∪ K b) H∩ K c) H+K d) None of these

9) The Quotient group of cyclic group is …….

a) cyclic b) abelian but not cyclic

c) neither abelian nor cyclic d) abelian

10) If N and M be two normal subgroup of a group G then NM is also ……. of G.

a) Normal b) Normal subgroup c) abelian d) cyclic

11) An one-one homomorphism is called……..

a) epimorphism b) monomorphism c) endomorphism d) automorphism

12) The order of symmetric group S_3 is ……

a) 3 b) 3^2 c) 6 d) 1

13) Every finite group is isomorphic to ……

a) a permutation group b) quotient group of G

c) a cyclic group d) none of these

14) A mapping f: G→ G' such that f(ab) = f(a) f(b) is called……

a) Homomorphism b) isomorphism
c) monomorphism d) epimorphism

15) "Every finite group G is isomorphism to a permutation group" is the statement of..

a) cauchy's theorem b) Euler's theorem
c) Leibnitz's theorem d) cayle's theorem

16) A commutative divison ring is called …..

a) vector space b) group c) Integral domain d) field

17) Every integral domain is not a ……

a) commutative ring b) integral domain c) field d) none of these

18) Which of the following statement is false ?

a) Every field is an integral domain.
b) Every finite integral domain is a field.
c) Every field is ring.
d) Every integral domain is a field.

19) Which of the following are zero divisors in a ring ($\mathbb{Z}_{18}$, $\oplus_{18}$,$\odot_{18}$).

a) 6, 3 b) 9, 2 c) 6, 4 d) both a) and b)

20) Which of the following is not an integral domain ?

a) ($\mathbb{Z}$, +, ·) b) ($2\mathbb{Z}$, +, ·) a) ($\mathbb{R}$, +, ·) a) ($\mathbb{R} \times \mathbb{R}$, +, ·)

Que. Long Answer Questions.

1) State and prove the Lagrange's Theorem.
2) If G= Z is the additive group of integers and H = 5Z is the subgroup of integers multiples of 5, then find [Z: 5Z].
3) Prove that, A subgroup H of a group G is normal in G iff the product of any two right cosets H in G is again a right coset of H in G.
4) Define Center of group. Prove that, Z(G) of group G is a normal subgroup of G.
5) Prove that, Every factor group of cyclic group is cyclic.
6) State and prove Fundamental theorem of group homomorphism.
7) Prove that, every finite group G is isomorphic to a permutation group.
8) Show that, a homomorphism f: G→ G' is one-one iff ker f = {e}, where e is identity element of G.
9) Define integral domain. Show that, A commutative ring R is an integral domain iff for all a, b, c$\in R, ab = ac, a \neq 0 \Rightarrow$ b= c.
10) Define subring. Show that, non-empty subset S of ring R is said to be subring of R if S forms ring under binary operations of R.
11)Let (Z, +) and (E,+) be the group of integers and the group of even integers respectively. Define a function f: Z→ E by f(x) = 2x for all x ∈Z. Prove that f is an isomorphism.

Que. Short Answer Questions.

1) State and prove Euler's theorem.

2) Prove that, If G is a finite group and a$\in G$, then o(a) divides o(G).
3) Show that, If a is any integer and p is prime, then $a^p \equiv a$ (mod p).
4) Prove that, If G be the group and H, K are two normal subgroups of G then H∩K is also normal subgroup of G.
5) Show that, the Center Z(G) of group G is normal subgroup of a normalizer of element.
6) If H is a normal subgroup of group G and K is any subgroup of group then HK =KH.
7) Define normal subgroup and prove that, a subgroup H of group G is normal iff ghg^{-1}= H, for all g∈ G.
8) Show that, every subgroup of abelian group is abelian.
9) Let, f be mapping from (Z, +) the group of integers to the group G = {1, -1} under multiplication defined as,

f(x) = 1 , if x is even
= -1 , if x is odd

Then show that f is homomorphism, Is it an isomorphism?
10) Prove that, If f: G→ G' be homomorphism defined from a group G into group G'and H be subgroup of G, then f(H) is a subgroup of G'.
11) Prove that, If f: G→ G' is an isomorphism and a is any element of G then o(f(a)) = o(a).
12) Find fg and gf if f = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 5 & 4 & 3 & 8 & 6 & 7 & 1 \end{pmatrix}$ and g = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 6 & 5 & 1 & 8 & 3 & 2 & 4 \end{pmatrix}$ and also find inverse of $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$.
13) If f = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$ and g = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ then show that fg ≠ gf.
14) Let M be the set of all 2× 2 matrices over integers. Let A ={ $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ / a, b are integers} then show that A is right ideal of M but not a left of M.
15) Consider a function f : ℂ → ℂ such that f(a+ib) = a-ib, a+ib∈ ℂ then show that f is homomorphism, where ℂ is the set of complex numbers.
16) Define Integral domain. Prove that, a commutative ring R is an integral domain iff for all a, b, c ∈ R. ab = ac, a≠0 ⇒ b = c.
17) Let R be the ring of all 2× 2 matrices over integers. Then show that set of matrices of the type A ={ $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ / a, b are integers} is a subring of R, but it is neither a left nor a right ideal of R.
18) In a ring R the following results hold: (i) $a \cdot 0 = 0 \cdot a = 0$ for all $a \in R$
(ii) $a \cdot (-b) = (-a) \cdot b = -ab$ for all $a, b \in R$ (iii) $a \cdot (b - c) = a \cdot b - a \cdot c$ for all $a, b, c \in R$