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B.Sc.I Paper DSC -6A

Calculus

Q. 1) Select correct alternatives of the following .

1. The f(x) and g(x) are function such that f(a)=0 and g(a)=0 then $\lim_{x\to a} \frac{f(x)}{g(x)}$ = …….

   a) $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ b) $\lim_{x\to a} \frac{g'(x)}{f'(x)}$ c) $\frac{f(a)}{g(a)}$ d) $\frac{f'(a)}{g(a)}$

2. A function $f(x)$ is said to be continuous at x= a if

   a) $\lim_{x\to a} f(x)$ exists. b) f(a) exists.

   c) $\lim_{x\to a} f(x) = \text{f(a)}$ d) $\lim_{x\to a} f(x) \neq \text{f(a)}$

3. The Taylor's series expansion of f(a+h) in ascending power of h is ……..

   a) $\text{f(a)} + \text{hf}'(\text{a}) + \frac{h^2}{2!}\text{f}''(\text{a}) + \cdots$ b) $\text{f(h)} + \text{af}'(\text{h}) + \frac{a}{2!}\text{f}''(\text{h}) + \cdots$

   c) $\text{f}(0) + \text{hf}'(0) + \frac{h^2}{2!}\text{f}'(0) + \cdots$ d) none of these

4. A polynomial function in IR ……..

   a) is never continuous in IR
   b) is always continuous in IR
   c) may or may not continuous in IR
   d) is continuous in IR except at x=0

5. $\lim_{x\to 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$ =………

   a) log 2 b) $\frac{1}{2}$log 2 c) 0 d) 2 log 2

6. If f(a) exists and $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$ both exists but are not equal then f has a ……… at x=a.

   a) jump discontinuity b) removable discontinuity

   c) discontinuity of second kind d) none of these

7. Roll's theorem is not applicable for function $f(x) = x^2$ in [0, 2] since

   a) $f(x)$ is continuous in [0, 2] b) $f(x)$ is not continuous in [0, 2]

   c) $f(0) \neq f(2)$ d) none of these

8. The geometrical meaning of Roll's theorem is that the tangent at point $c \in (a, b)$ is

   a) parallel to y axis b) parallel to x axis

   c) intersecting to x and y axis d) none of these

9. The geometrical meaning of lagrange mean value theorem is that the tangent at point $c \in (a, b)$ is

a) perpendicular to chord AB   b) parallel to chord AB
c) intersecting to chord AB   d) none of these

10. The function $f(x) = \begin{cases} 2x - 1 & 1 \le x < 2 \\ 1 + x & 2 \le x \le 3 \end{cases}$ is

a) continuous at x=2   b) f (2) does not exists.
c) discontinuous at x=2   d) none of these.

11. Roll's theorem is not applicable for function $f(x) = sinx$ in $[0, \frac{\pi}{2}]$ since

a) $f(x)$ is not continuous at x=0   b) $f(0) \neq f\left(\frac{\pi}{2}\right)$
c) $f(x)$ is not differential at x=0   d) $f(x)$ is not continuous in $[0, \frac{\pi}{2}]$

12. The expansion of e^x is

a)$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ b) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$
c) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ d) $1 + x + x^2 + x^3 + \cdots$

13. The function $f(x) = \frac{x^2-1}{x-1}$ has

a) jump discontinuity at x=1   b) removable discontinuity at x=1
c) infinite discontinuity at x=1   d) none of these

14. The expansion of sin x is

a)$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ b) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$
c) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ d) $1 + x + x^2 + x^3 + \cdots$

15. $\lim_{x \to \frac{\pi}{2}} \frac{1 - sinx}{cosx} = \cdots$

a)1   b)0   c)$\frac{1}{2}$ d)-1

16. $\lim_{x \to \pi} \frac{sinx}{x - \pi} = \cdots$

a)1   b)0   c)2   d) -1

17. $\lim_{x \to 1} \frac{1 - e^{x-1}}{x - 1} = \cdots$

a)2   b)0   c)1   d)-1

18. $\lim_{x \to 0} xlogx = \cdots$

a)2   b)-1   c)1   d)0

19. The function $f(x) = |x|$ at x=0 is

a) not continuous   b) differentiable

c) not differentiable d) none of these.

20. The Maclaurin series expansion of f(x) in ascending power of x is ……..

a) $f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \cdots$ b) $f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots$

c) $f(0) + f'(0) + \frac{f''(0)}{2!} + \cdots$ d) none of these

21. If f(x) satisfies all condition of Roll's mean value theorem in $[a, b]$ then there exist a point $c \in (a, b)$ such that

a) $f(a) = f(b)$ b) $f'(c) > 0$ c) $f'(c) = \frac{f(b)-f(a)}{b-a}$ d) $f'(c) = 0$

22. $\lim_{x \to \frac{\pi}{2}} (1 - \sin x)\tan x = \cdots$

a)1 b) -1 c)π d)0

23. $\lim_{x \to 0^-} \frac{|x|}{x} = \cdots$

a)1 b) -1 c) 0 d) 2

24. $\lim_{x \to 0} \frac{\sin\frac{x}{2}}{x} = \cdots$

a)1 b) 2 c)$\frac{1}{2}$ d) not defined

25. The function $f(x) = \sin x$ at x=0 is

a) not continuous b) differentiable

c) not differentiable d) none of these

Q. Long and short answer.

1) Verify Rolle's theorem for the function $f(x) = x^2 - 3x + 2$ in [1, 2].

2) Verify Lagrange's mean value theorem, for f(x) = log x in [1, e]

3) Evaluate following limit, $\lim_{x \to 0} \frac{x - \sin x}{x^2}$.

4) Applying Lagrange's M.V.T. show that $0 < \frac{1}{x} \log\left(\frac{e^x - 1}{x}\right) < 1$.

5) Evaluate $\lim_{x \to 1}\left(\frac{x}{x-1} - \frac{1}{\log x}\right)$

6) Using the definition, prove that $\lim_{x \to 3} \frac{3x-3}{2} = 3$.

7) Examine the continuity at the indicated point f(x) = x - |x| at x = 0.

8) Test the differentiability of $f(x) = x^2 \sin\frac{1}{x}$ for $x \neq 0$ and f(0) = 0.

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Class: B.Sc.-I

Differential Equation (DSC-5B)

QUESTION BANK

Q. Select the correct alternatives for each of the following.

1. The differential equation Mdx+Ndy=0 is exact if-----

   a) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial X}$ b) $\frac{\partial^2 M}{\partial x^2}=\frac{\partial^2 N}{\partial y^2}$ c) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ d) $\frac{\partial^2 M}{\partial y^2}=\frac{\partial^2 N}{\partial x^2}$

2. Which of the following differential equation is linear-----

   a) $dy/dx+x^2y=\sin y$ b) $dy/dx-x^2y=x\sin x$

   c) $(y+1)dy/dx+\sin x=x^2$ d) $dy/dx+y(x+y)=x^3$

3. The differential equation of the form $dy/dx+P(x)y=Q(x)y^n$ is called ------

   a)Euler's equation b)linear equation c)Bernoulli's equation d)Clairauts equation

4. The integrating factor of dy/dx +Py=Q is-----

   a) $e^{\int pdy}$ b) $e^{\int -fpdx}$ c) $e^{\int pdy}$ d) $e^{\int -pdy}$

5. The equation $(x^2-ay)dx+(y^2-ax)dy=0$ is ----

   a)Homogeneous b)variables separable c)Exact d)Linear

6. The solution of the differential equation y= px+f(p) is----

   a)y=cx-f(c) b)y=cx+f(x) c)y=cx+f(c) d)y=px+f(c)

7. The solution of the equation $p^2-7p+12=0$ is -----

   a)(y-3x-c)(y-4x-c)=0 b)(y-6x-c)(y-3x-c)=0 c)(y+3x+c)(y-4x-c)=0 d) y=2x+c

8. The general solution of Clairauts equation represent a family of -----

   a)hyperbola b)circles c) ellipses d)straight lines

9. The equation of the form y=px+f(p) is ----

   a)linear b)Bernoulli's c) Clairaut's d)DeMoivre's

10. The value of $1/D^2+1 \cos x$ is ----

    a)xcosx b)xsinx c) $\frac{x}{2}\cos x$ d)-xcosx

11. The particular integral (P.I) of $(D-a)^2y=e^{ax}$ is ----

a)$a^2\, e^{ax}$ b)$x^2/2\, e^{ax}$ c) x^2e^{ax} d)$x/2\, e^{ax}$

12. The value of $1/f(D)\ (e^{ax})$ =----

a) $e^{ax}1/f(D$ b) $1/f(D-a)\ V$ c) $1/f(a)\ e^{ax}$ d)None of these

13. The P.I. of $(D-1)^2y = x$ is -----

a)$x- x^2$ b)$x+2$ c) x^2+2 d)$1-x$

14. The C.F. of the equation $d^2y/dx^2-3dy/dx+4y=e^{2x}$ is ---- -

a)$y=c_1e^{x} +c_2e^{2x}$ b) $y=c_1e^{x} +c_2e^{-2x}$ c) $y=c_1e^{-x} +c_2e^{-2x}$ d) $y=(c_1x +c_2)e^{2x}$

15. The homogeneous linear equation can be reduced to linear equation with constant coefficient by using the substitution---

a)$x=e^z$ b)$z=e^x$ c) $x=e^{-z}$ d)$z=e^{-x}$

16. The differential equation $d^2y/dx^2+\frac{1}{x2}\,y=1$ is ------equation

a) homogeneous linear b) non-homogeneous
c) linear equation with constant coefficient d) exact differential

17. By using substitution z=logx , a homogeneous linear differential equation can be reduced to---

a)Second order linear equation b) Linear differential equation with constant coefficients
c)Exact differential equation d) Total differential equation

18. By using substitution $x=e^z$ the value of $x^3\, d^3y/dx^3$ in the homogeneous linear equation is ----

a)$(D-1)(D-2)(D-3)$ b)$D(D-1)(D-2)$ c)$D^2(D-1)$ d)D

19. The solution of homogeneous linear equation $x^2\, d^2x/dx^2 -x\, dy/dx -3y=0$ is ----

a)$y=c_1x^3+c_2/x$ b) $y=c_1x^3+c_2\, x$ c) $y=c_1\, e^{-3x} +c_2\, e^{-x}$ d) $y=c_1x+c_2/x^3$

20. By using substitution $x=e^z$,the value of $x^2\, d^2y/dx2$ in the homogeneous linear equation is ----

a)$(D-1)(D-2)$ b)$D(D-1)$ c)$D(D+1)$ d)D^2

21. The differential equation $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial y}}{N}$ =f(x)then corresponding integrating factor is ------

a)$e^{\int f(x)dy}$ b) $e^{\int f(x)dx}$ c) $e^{\int f(y)dx}$ d) $e^{\int f(y)dx}$

22. $\frac{1}{D2+a2}\ sinax$ =-----

a) $-\frac{x}{2a}cosax$ b) $-\frac{x}{2a}sinax$ c) $-\frac{x}{2a}sinax$ d) $\frac{x}{2a}cosax$

23. To solve $(px-y)(x-py)=p$, by using substituting -----

a) $x^2 = u$ and $y^2 =v$ a) $x^2 =v$ and $y^2 =u$ c) $x^2 = u$ and $y =v$ d) $x = u$ and $y^2 =v$

24. $\frac{1}{D}\, x$ =---

a) $\int xdx$ b) $\int ydx$ c) $\int xdy$ d) $\int ydy$

25. The value of $1/f(D^2)$ sinax=------

a) $1/f(a^2)$ sinax b) $1/f(-a^2)$ sinax c) $1/f(a^2)$ cosax d) $1/f(-a^2)$ cosax

Q. Long Answer Type question.

1) State and prove the necessary and sufficient condition for differential equation Mdx+Ndy = 0 to be exact.

2) Solve $p^2+2py\cot x = y^2$

3) Prove that, The differential operator D= $\frac{d}{dx}$ processes the following property,

$(D-\alpha)(D-\beta)\, y = (D-\beta)(D-\alpha)\, y$

4)Evaluate $\frac{1}{D+2}$ sinx

5) Define Bernoulli's differential equation and explain the method for solving it.

Solve $(1-x^2)\frac{dy}{dx} + xy = xy^2$.

6) Solve $(D^3-3D^2+4D-2)y=e^x+\cos x$

7) Solve $(x^3y^3+x^2y^2+xy+1)y\,dx + (x^3y^3-x^2y^2-xy+1)x\,dy = 0$

8) Solve $(3x+2)^2 \frac{d^2y}{dx^2} +3(3x+2)\frac{dy}{dx} -36y = x^2+x+1$

9) Explain method for solving Linear differential equation.

10) Define Clairaut's Equation and explain method for solving it. Solve $(y-px)^2/(1+p^2) = a$

Q. Short Answer Type Question.

1) Solve $x(x-1)\frac{dy}{dx} - (x-2)y = x^3(2x-1)$.

2) Solve $[\sin x\cos y + e^{2x}]\,dx + [\cos x\sin y + \tan y]\,dy=0$

3) Solve $(y-2x^2)\,dx - x(1-xy)\,dy = 0$

4) Solve $y(xy+2x^2y^2)\,dx+ x(xy-x^2y^2)\,dy=0$ by using rule for finding I.F.

5) Solve $xp^2-2yp+x = 0$

5) Solve $x = y+p^2$

6) Solve $y^2(y - px) = x^4 p^2$ by using substitution $x=\frac{1}{u}$ $y=\frac{1}{v}$

7) Solve $\frac{d^4y}{dx^4} - m^4y = 0$

8) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 2x$

9) Solve $\frac{d^2y}{dx^2} -5\frac{dy}{dx} +6y = 2e^{3x}$

10) Solve $(D^3-3D+2)\, y = x$

11) Solve $x^2\frac{d^2y}{dx^2} -4x\frac{dy}{dx} +6y= x$

12) Solve $p^2(x^2-a^2) - 2xyp + (y^2+a^4)=0$ by using clairaut's equation.

13) Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx} -4y= x^2$

14) Solve $\frac{1}{D^3} e^{3x}$

15) Evaluate $\frac{1}{(D-1)(D-2)} e^{3x}$

Answer key :- 1) a 2)a 3)c 4) b 5) c 6) a 7) a 8)d 9) c 10) a
11) b 12) c 13) a 14) d 15) a 16)c 17) b 18)b 19) c 20) b

21) b 22) a 23) a 24) a 25) b

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Class: B.Sc.-I (Sem. II) Paper: III
Multivariable Calculus (DSC-B5)
QUESTION BANK 2022-2023

Q. Select the correct alternatives for each of the following.

1. If A= $3x^2y-y^3$ then $\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = \ldots$
   a) $3x^2 - 3y^2$ b) 6xy c) 12y d) 0
2. The function u = $\sin\left(\frac{x^2+y^2}{x+y}\right)$ is …
   a) a homogenous function of degree 1. b) a homogenous function of degree 2.
   c) a homogenous function of degree 0. d) not a homogenous function.
3. If $u = x^2 + 2y^2 + 3z^2$, where $x = e^t$, $y = e^{2t}$, $z = e^{3t}$, then $\frac{du}{dt}$ at t = 0 is…
   a) 4 b) 10 c) 28 d) 0
4. If z = f(x,y) is a homogeneous function of degree n then, $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = \ldots$
   a) nz b) n(n-1) z c) n $\frac{f(u)}{f'(u)}$ d) g(u)[g'(u)-1]
5. If $z = \frac{xy}{x+y}$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \ldots$
   a) z b) 0 c) 2z d) z^2
6. If $G = \tan^{-1}\left(\frac{x}{y}\right)$ where x = u + v and y = u – v then $\frac{\partial G}{\partial u} = \ldots$
   a) $\frac{u}{u^2+v^2}$ b) $\frac{-v}{u^2+v^2}$ c) $\frac{u}{x^2+y^2}$ d) $\frac{-2v}{x^2+y^2}$
7. The degree of the homogeneous function $\frac{x+y}{\sqrt{x}+\sqrt{y}}$ is …
   a) $\sqrt{2}$ b) 1 c) $-\frac{1}{2}$ d) $\frac{1}{2}$
8. If $u = \sin(x^2y^2)$ then $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 \ldots$
   a) $4x^2y^2\cos(x^2y^2)$ b) $4x^2y^2(x^2 + y^2)$ c) $4x^2y^2(x^2 + y^2)\cos(x^2y^2)$ d) none of these
9. If $f(x,y) = x^3 + y^3 - 2x^2y^2$, then $f_{xx \mid x = y = 1} = \ldots$
   a) 2 b) -2 c) 1 d) -1
10. If $u = \log\left(\frac{x^5+y^5}{x+y}\right)$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \ldots$
    a) 2 b) 4 c) 6 d) -2
11. If $u = e^x \cos y$, $v = e^x \sin y$, then the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ is …
    a) e^x b) 0 c) 1 d) $e^x \sin y \cos x$
12. If $u = x^2$, $v = y^2$ then the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ is…
    a) 4xy b) -4xy c) $4x^2y^2$ d) $\frac{4x}{y}$
13. If x = u+v, y = u-v then the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ is…

a) 0 b) 1 c) 2 d) -2

14. If x = uv+u, y = uv-v then then the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ is…

a) 2u b) u + v c) -2v d) u – v

15. A function f(x, y) has an extreme value at (a, b) then…

a) $AC - B^2 > 0$ b) $AC - B^2 < 0$ c) $AC - B^2 = 0$ d) $AB - C^2 > 0$

16. A function f(x, y) has extreme value at (a, b) then …

a) $f_x(a, b) > 0$ b) $f_y(a, b) > 0$ c) $f_x(a, b)\, f_y(a, b) = 0$ d) $f_x(a, b) = f_y(a ,b) = 0$

17. The Lagrange's method of undetermined multipliers is used for finding the extreme value of …

a) one variable b) two variable c) three or more variable d) none of these

18. A function f(x, y) is maximum at a point (a, b) if …

a) $AC - B^2 > 0$ and $A > 0$ b) $AC - B^2 > 0$ and $A < 0$
c) $AC - B^2 < 0$ and $A > 0$ d) $AC^2 - B^2 < 0$ and $A< 0$

19. A function f(x, y) is minimum at a point (a, b) if …

a) $AC - B^2 > 0$ and $A > 0$ b) $AC - B^2 > 0$ and $A < 0$
c) $AC - B^2 < 0$ and $A > 0$ d) $AC^2 - B^2 < 0$ and $A< 0$

20. If $f(x, y, z) = x^2 + xyz + z$, then $f_x(1, 1, 1) = \ldots$

a) 0 b) 3 c) 1 d) -1

Q. Long Answer Type Question

1. State and prove Euler's theorem on homogeneous function.
2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, if $x^3 + y^3 = 3axy$.
3. Expand $f(x, y) = x^2 + xy - y^2$ in power of (x – 1) and (y + 2)
4. If $z = \tan^{-1}\left(\frac{x}{y}\right)$, x = u + v and y = u-v, show that $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = \frac{u\, v}{u^2+v^2}$
5. Define: i) neighbourhood of point ii) limit of function of two variables iii) continuity iv) statement of Euler's theorem.
6. Find the extreme values of the following function $u = x^2 + y^2 + 6x + 12$.
7. Prove that the stationary value of $u = \frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$ subject to the conditions lx + my + nz = 0 and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ are the roots of the equation $\frac{l^2a^4}{1-a^2u} + \frac{m^2b^4}{1-b^2u} + \frac{n^2c^4}{1-c^2u} = 0$.
8. If $x^2 + y^2 + u^2 - v^2 = 0$ and $uv + xy = 0$. Show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{x^2-y^2}{u^2+v^2}$.
9. Let u, v be functions of two variable x, y. If the Jacobian $J = \frac{\partial(u,v)}{\partial(x,y)}$ and $J' = \frac{\partial(x,y)}{\partial(u,v)}$ then show that JJ' = 1.
10. If p, q, r are function of x, y,z and x, y, z are functions of u, v, w, then prove that $\frac{\partial(p,q,r)}{\partial(u,v,w)} = \frac{\partial(p,q,r)}{\partial(x,y,z)} \cdot \frac{\partial(x,y,z)}{\partial(u,v,w)}$.
11. For $x = \sqrt{vw}$, $y = \sqrt{uw}$, $z = \sqrt{uv}$, $u = r \sin\theta \cos\phi$, $v = r \sin\theta \sin\phi$, $w = r\cos\theta$ find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$
12. If x = a(u + v), y = b(u – v) and $u = r^2\cos 2\theta$, $v = r^2\sin 2\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

13. Let u, v, w be functions of three variables x, y, z. Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} \cdot \frac{\partial(x,y,z)}{\partial(u,v,w)} = 1$

Q. Short Answer Type Questions.

1. Determine whether f(x, y) defined below has a limit as (x, y) → (0, 0).

$$f(x, y) = \frac{xy^2}{x^2+y^4} \quad \text{if } (x, y) \neq (0, 0)$$
$$= 0 \quad \text{if } (x, y) = (0, 0)$$

2. Discuss the continuity of $f(x, y) = \frac{\sqrt{xy}}{x+y}$; $(x, y) \neq (0, 0)$

$$f(x, y) = 0$$

3. Find $\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial x}$, where u(x, y) = y sin xy

4. If f(x, y) = x^2 cos y + y^2 sin x, find f_x, f_y, f_{xx}, f_{xy}, f_{yy}.

5. If u = log ($x^2 + y^2$), show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

6. If z = x^2y^2 + x + y and x = t^2, y = 2t then find $\frac{dz}{dt}$

7. Verify Euler's theorem for z = $ax^2 + 2hxy + by^2$

8. Differentiate f(x, y) = sin y + x^2 + 4y = cos x w. r. t. x and find $\frac{dy}{dx}$.

9. Expand e^{ax} sin by in power of x, y as far as third degree.

10. Find the points on the surface z^2 = xy + 1, which are at the least distance from the origin.

11. Explain Lagrange's method of undermined multipliers to find extreme values of function of three variable subject to the one condition.

12. If D = $\sqrt{x^2 + y^2 + z^2}$ and x + y + z = 30, find the value of x, y, z for which D is minimum.

13. Find extreme value of xy when $x^2+xy+y^2 = a^2$.

14. Find the stationary value of $x^2 + y^2 + z^2$ subject to the condition lx + my + nz = p and interpret the result.

15. If x = r cosθ, y = r sin θ, compute J = $\frac{\partial(x,y)}{\partial(r,\theta)}$ and J'= $\frac{\partial(r,\theta)}{\partial(x,y)}$ and verify that, JJ' = 1.

16. Find the Jacobian of the following transformation x = r sin θ $cos\phi$, y = r sin θ $sin\phi$, z = r cos θ.

17. If u = $x^2 - y^2$, v = xy, find $\frac{\partial(u,v)}{\partial(x,y)}$.

18. If $u^3 + v^3$ = x + y and $u^2 + v^2 = x^3 + y^3$. Find $\frac{\partial(u,v)}{\partial(x,y)}$.

19. If z = $x^2y + y^3$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

20. If z = $\frac{x^2+y^2}{x+y}$ show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$.

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B.Sc. I Paper DSC- 6A (Basic Algebra)

Question Bank (2022-23)

Q. Multiple Choice Questions.

1) If $A = \{1,2,\{3\},\{4,5\}\}$ then number of element in it is…..

a) 3 b) 4 c) 5 d) 2

2) A relation on set which is reflexive, anti-symmetric & transitive is called…..

a) Irreflexive relation b) an equivalence relation
c) partial order relation d) symmetric relation

3) If n is even then $n =$….. for some $k \epsilon Z$

a) 2k b) k-1 c) 2k-1 d) 2k=1

4) If $n|a - b$,then…..

a) $a \equiv b(mod\ n)$ b) $n \equiv a(mod\ b)$ c) $a \equiv 1(mod\ n)$ d) $n \equiv b(mod\ a)$

5) ….. is the remainder of 12^{12} when divided by 13.

a) 12 b) 3 c) 1 d) 0

6) If $x = cos\theta + isin\theta$, then $x^n + \frac{1}{x^n}$ is…..

a) $2\sin(n\theta)$ b) $2\cos(n\theta)$ c) $2(cos\ n\theta + isin\ n\theta)$ d) none of these

7) The value of i^i is ……

a) $\frac{\pi}{2}$ b) $\frac{-\pi}{2}$ c) $e^{\frac{\pi}{2}}$ d) $e^{\frac{-\pi}{2}}$

8) If $z = -1 + i$, then using DeMoivre's theorem $z^4 =$…..

a) -4 b) 4 c) 4i d) none of these

9) If F:R →R is defined as f(x)= x then f is -----

a) Injection b) Surjection c) Bijection d) None of these

10) ….. is the remainder of 12^{12} when divided by 13.

a) 12 b)3 c) 1 d) 0

11) The relation R is called equivalence relation if the relation is -----

a) reflexive, symmetric & transitive b) Transitive
c) partial order relation d) symmetric relation

12) If $sinhz = i$, then …..

a) $x = 0, y = 0$ b) $z = \frac{\pi}{2}, y = 0$
c) $x = 0, y = \frac{\pi}{2}$ d) $x = \frac{\pi}{2}, y = \frac{\pi}{2}$

13) Using DeMoivers theorem, $\frac{(1+i\sqrt{3})^6}{(1-i\sqrt{3})^6} = \cdots$

a) $e^{-i\pi}$ b) $e^{i4\pi}$ c) $e^{\frac{i\pi}{2}}$ d) $e^{\frac{i3\pi}{2}}$

14) The value of $\tanh\left(log\sqrt{3}\right) = \cdots$

a) 1 b) 2 c) $\frac{1}{2}$ d) none of these

15) If $y = sinhx$ then $\frac{dy}{dx} = \cdots$

a) $sinhx$ b) $coshx$ c) $-sinhx$ d) $-coshx$

16) If p is a prime, which does not divide the integer a then g.c.d. of p and $a = \cdots$

a) a b) p c)1 d) 0

17) $\emptyset(13) = \cdots$ where $\emptyset$ is Euler's $\emptyset$ function.

a) 9 b) 12 c)10 d) 11

18) If F:R →R is defined as f(x)= x^2+1, then $f^{-1}(-5)$ is …..

a) -5 b) 5 c) $\emptyset$ d) 0

19) If set X contains m elements and Y contains n elements, then X×Y contains ….. elements.

a) m - n b) m+n c) mn d) $(mn)^2$

20) If $g(x) = x^2$ $(0 \leq x < \infty)$, then $g^{-1}(x)$ = …..$(0 \leq x < \infty)$

a) x^2 b) x c) $x^{3/2}$ d) $x^{1/2}$

Q. 2) Long & Short Questions

1) Define Composition of Function. and Prove that, The Composition of two bijective function is bijective.

2) Find all the values of $(1 + i\sqrt{3})^{3/4}$

3) Define Principle of Mathematical Induction. & Prove that,

$$1 + 2 + \cdots + n = \frac{1}{2}n(n + 1).$$

4) Simplify $\frac{(\text{Cos}2\theta - \text{isin}2\theta)5\ (\text{Cos}3\theta + \text{isin}3\theta)-4}{(\text{Cos}3\theta + \text{isin}3\theta)-2\ (\text{Cos}5\theta - \text{isin}5\theta)3}$

5) State and prove Demovrie´s theorem

6) Define Types of Relations.

7) Evaluate, $\int cos^5 \theta\ d\theta$.

8) If $A = \{x \epsilon \mathbb{R} | x \neq 1\}$ and define $f(x) = \frac{2x}{x-1}; \forall\ x \epsilon A$ then show that f is injective and find range of f.

9) Define Hyperbolic function for cosine and state its fundamental formulaes.

10) Find the remainder when 12^{12} is divided by 13.

11) If $f(n) = n + 7$ & $(x) = 2n$; $n \epsilon \mathbb{Z}$ then find value of $f{\circ}g$ and $g{\circ}f$.

12) Prove that tan-1^x=1/2 log ($\frac{1+x}{1-x}$)

13) Define Refiexive ,Symmetric ,Transitive Relations.

14) Evaluate,$(\sqrt{3+i)})^n$+$(\sqrt{3-i)})^n$ =2n+1 cos $\frac{n\pi}{6}$

15) For all rational numbers $(\text{COS}\theta+\text{isin}\theta)^n$ has one of the value cosnθ+isin nθ i.e.

16) Prove that $\sin^{-1}hx$ = log (x+ $\sqrt{x2+1}$)

17) Prove that, $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$for each $n \in N$.

18) Find Remainder when 45^{65} is divided by 7.

19) Evaluate $Log(1+i)$.

20) Prove that $(1+i)^n + (1-i)^n = 2^{(\frac{n}{2}+1)}.cos\frac{n\pi}{4}$.

In Particular, $(1+i)^8 + (1-i)^8 = 32$

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