SCALARS
&
VECTORS

What is Scalar?

Length of a car is 4.5 m

physical quantity

magnitude

Mass of gold bar is 1 kg

physical quantity

magnitude

Time is 12.76 s

physical quantity

magnitude

Temperature is 36.8 °C

physical quantity

magnitude

A scalar is a physical quantity that has only a magnitude.

Examples:

- Mass
- Length
- Time
- Temperature
- Volume
- Density

What is Vector?

California

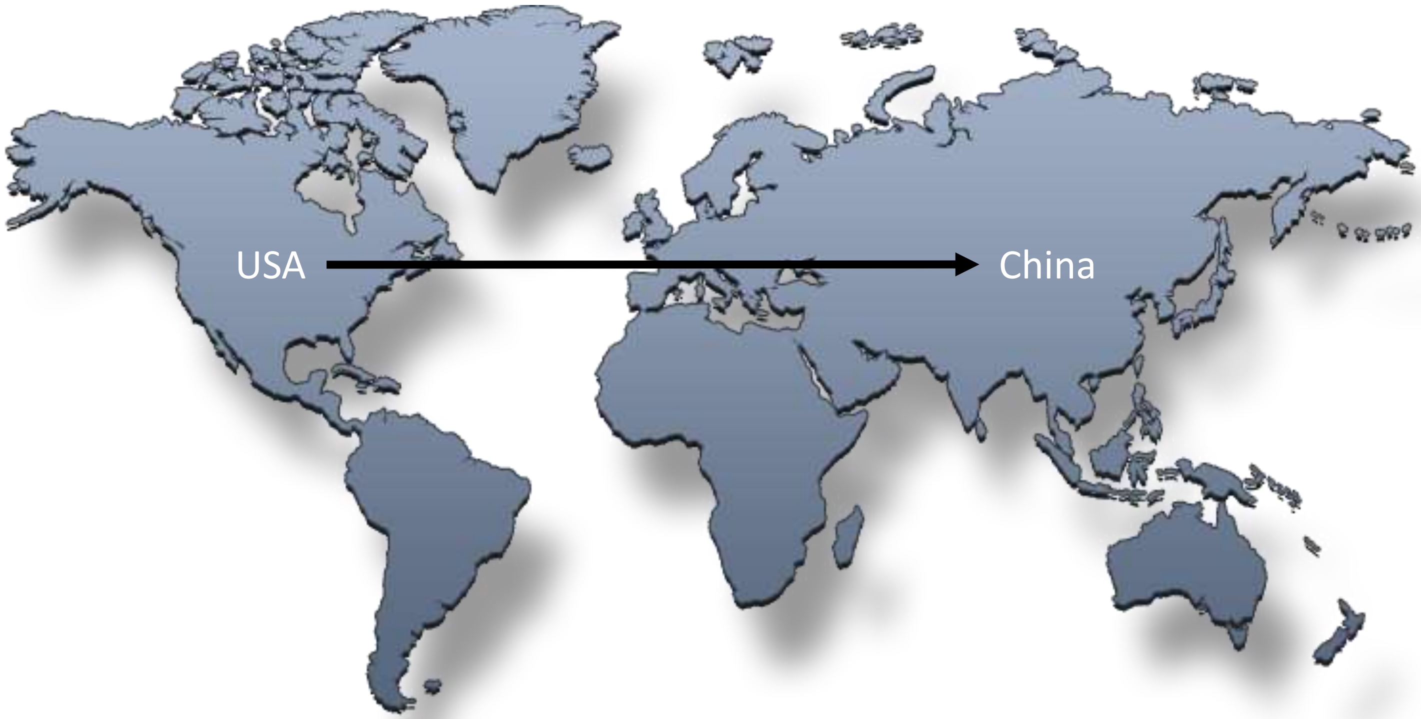
North Carolina

Position of California from North Carolina is 3600 km in west

physical quantity

direction

magnitude



Displacement from USA to China is 11600 km in east

physical quantity

direction

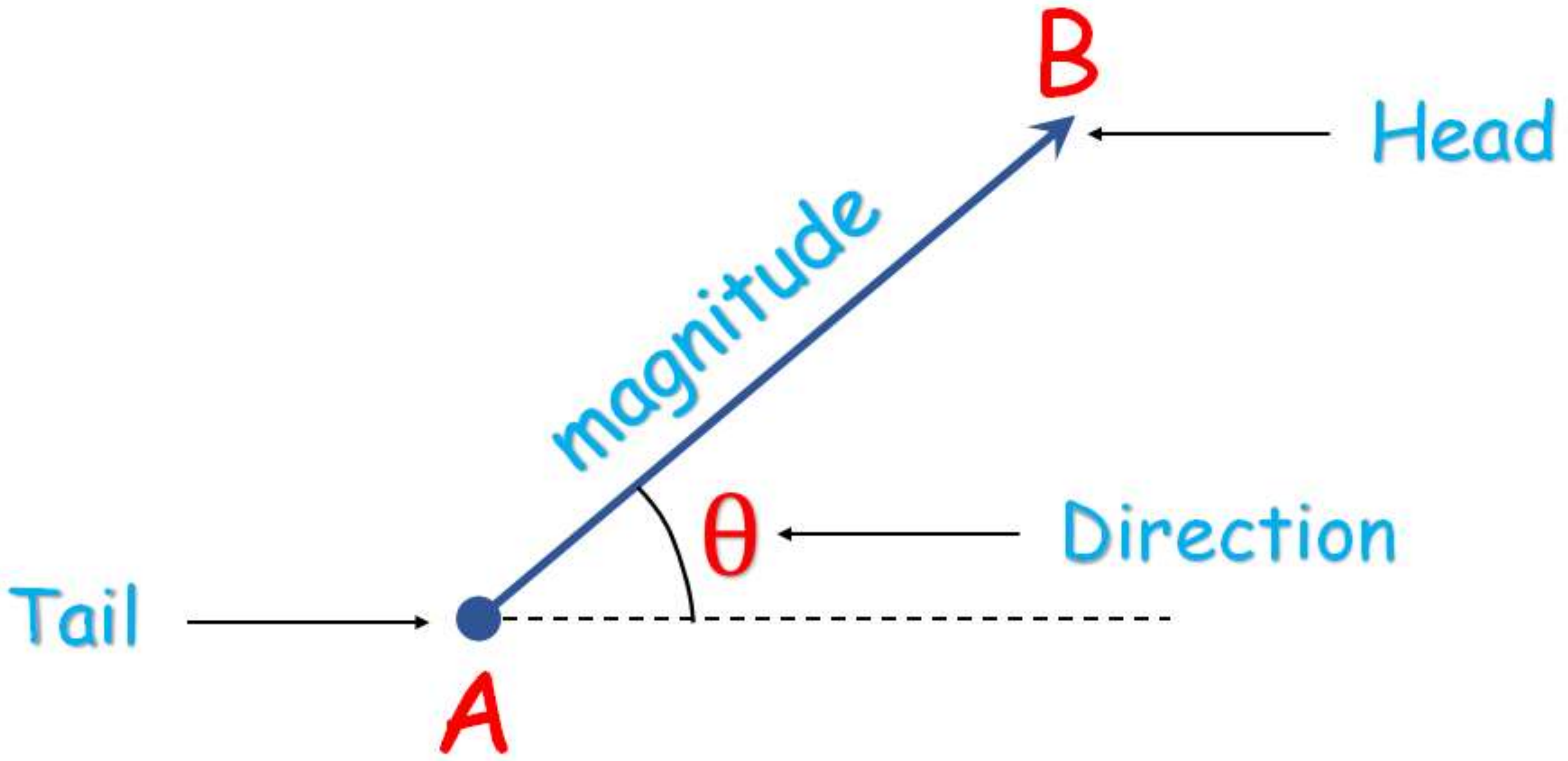
magnitude

A vector is a physical quantity that has both a magnitude and a direction.

Examples:

- Position
- Displacement
- Velocity
- Acceleration
- Momentum
- Force

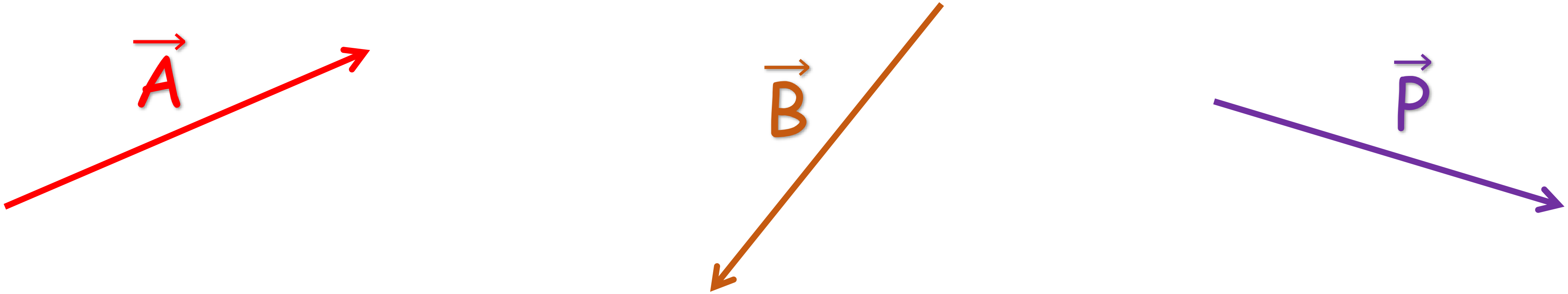
Representation of a vector



Symbolically it is represented as $\overrightarrow{AB}$

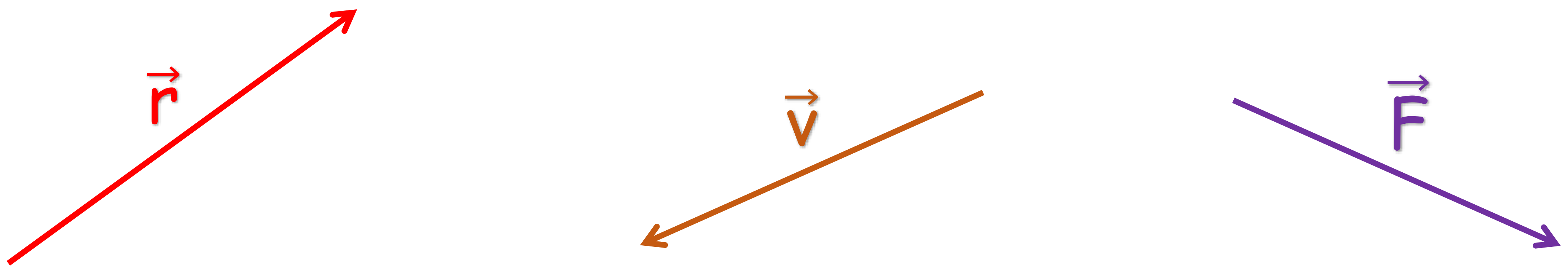
Representation of a vector

They are also represented by a single capital letter with an arrow above it.



Representation of a vector

Some vector quantities are represented by their respective symbols with an arrow above it.



Position velocity Force

Types of Vectors

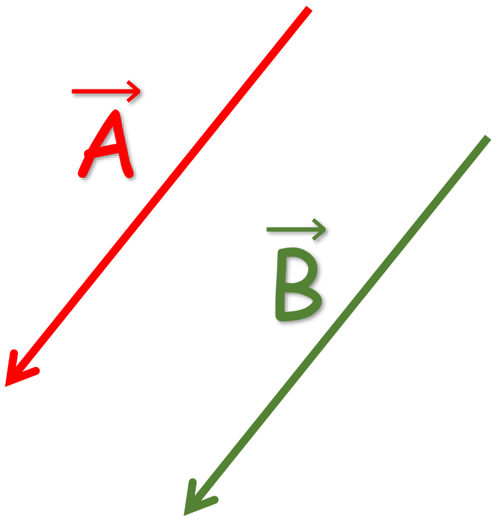
(on the basis of orientation)

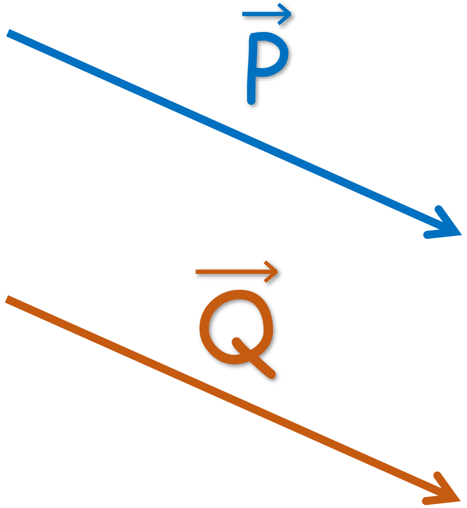
Parallel Vectors

Two vectors are said to be parallel vectors, if they have same direction.

Equal Vectors

Two parallel vectors are said to be equal vectors, if they have same magnitude.



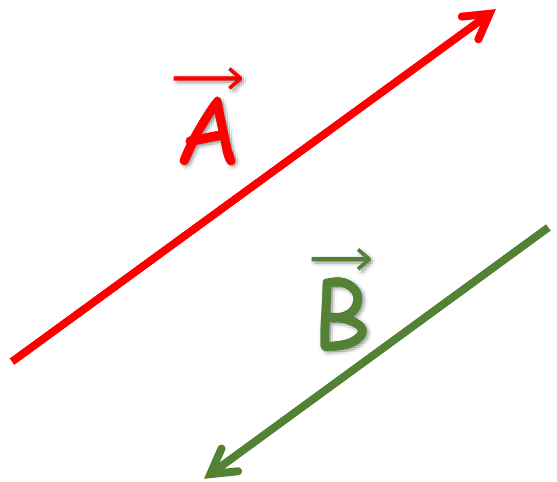


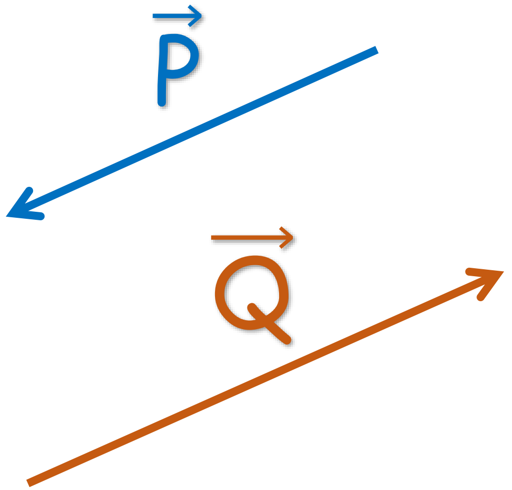
$$\vec{A} = \vec{B}$$

$$\vec{P} = \vec{Q}$$

Anti-parallel Vectors

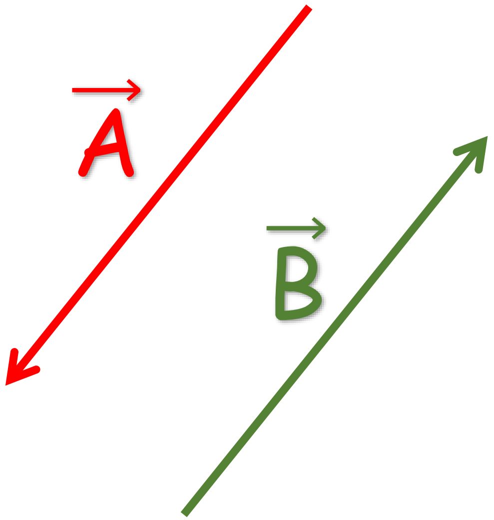
Two vectors are said to be anti-parallel vectors, if they are in opposite directions.



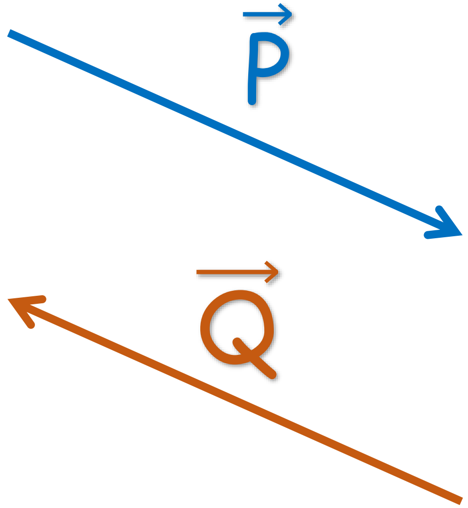


Negative Vectors

Two anti-parallel vectors are said to be negative vectors, if they have same magnitude.



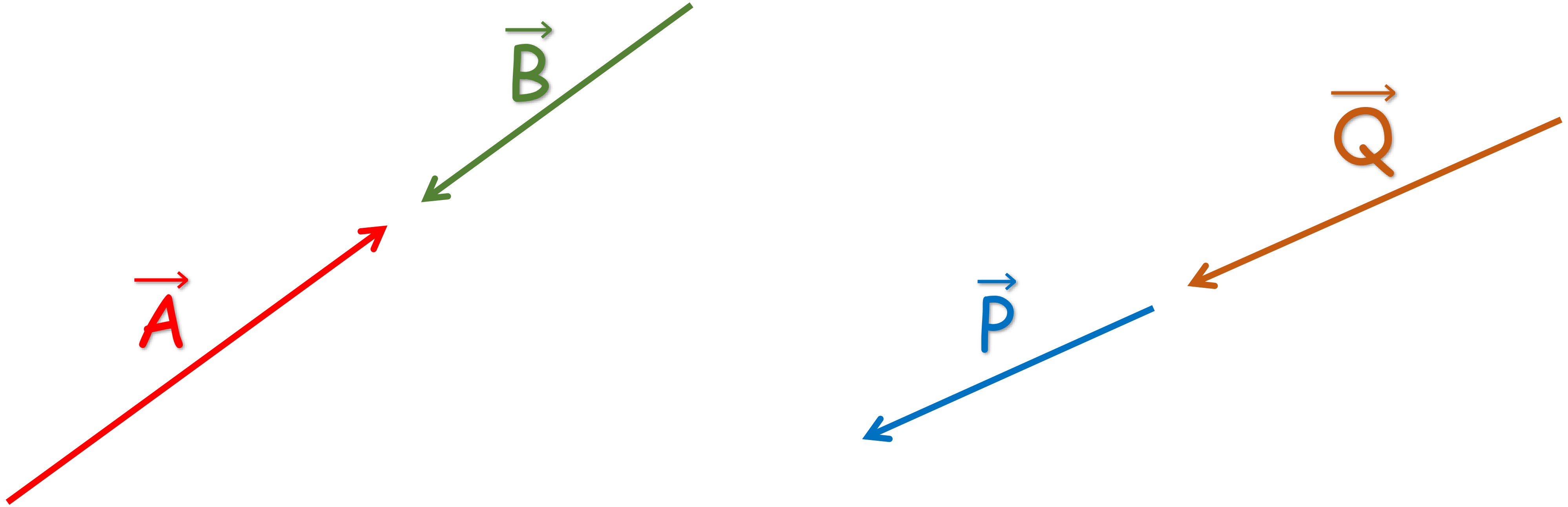
$$\vec{A} = -\vec{B}$$



$$\vec{P} = -\vec{Q}$$

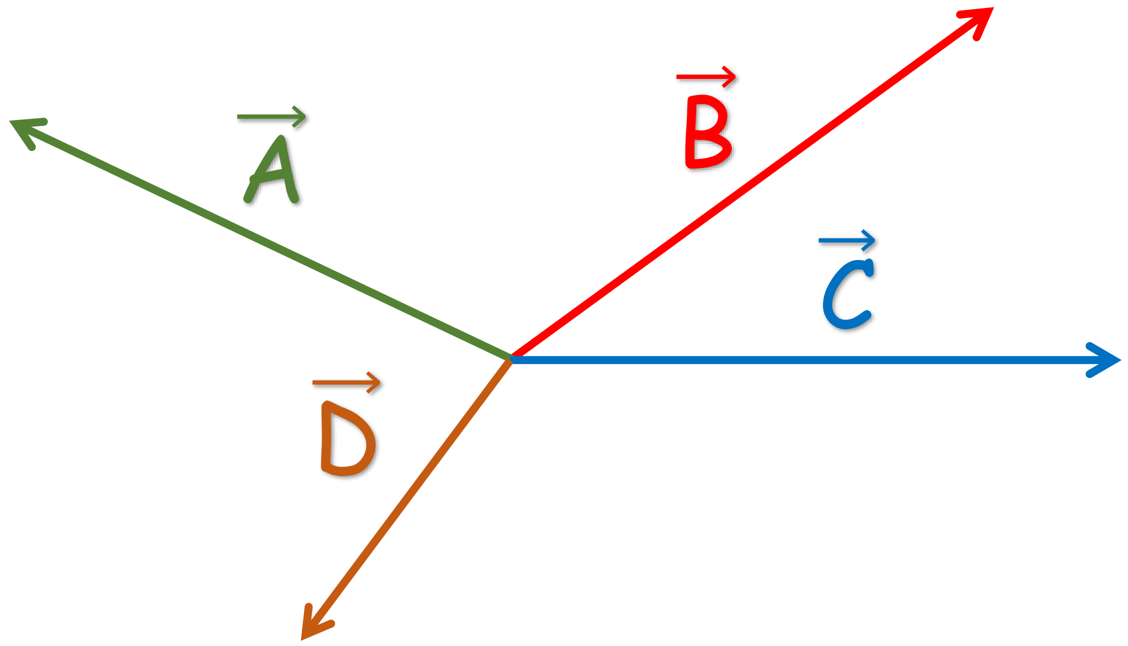
Collinear Vectors

Two vectors are said to be collinear vectors, if they act along a same line.



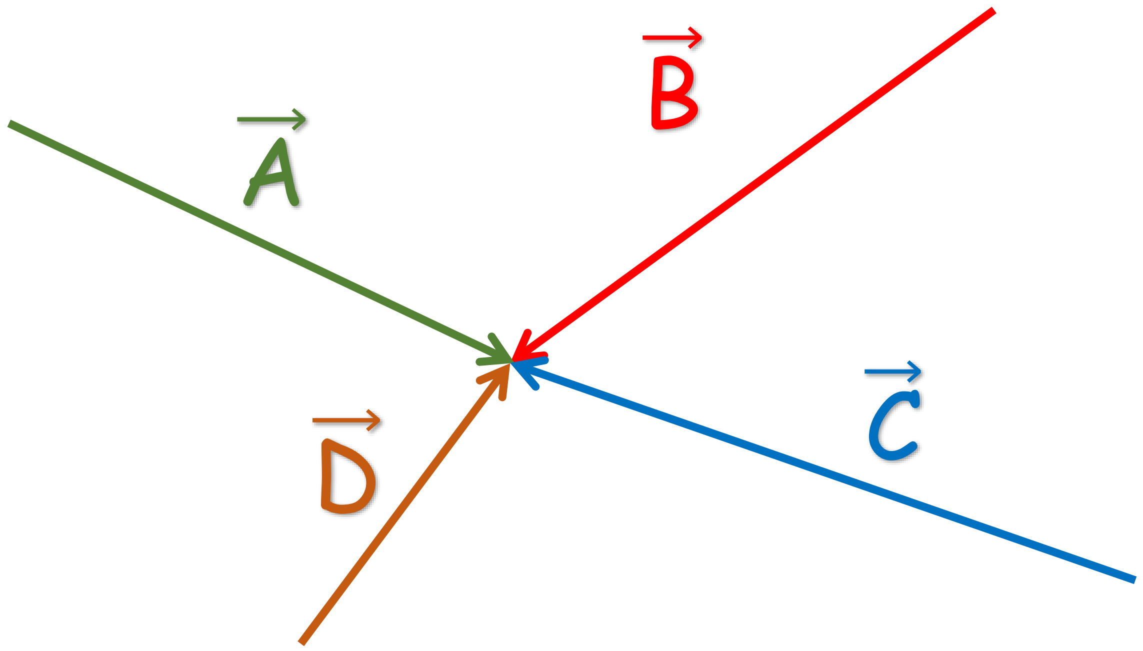
Co-initial Vectors

Two or more vectors are said to be co-initial vectors, if they have common initial point.



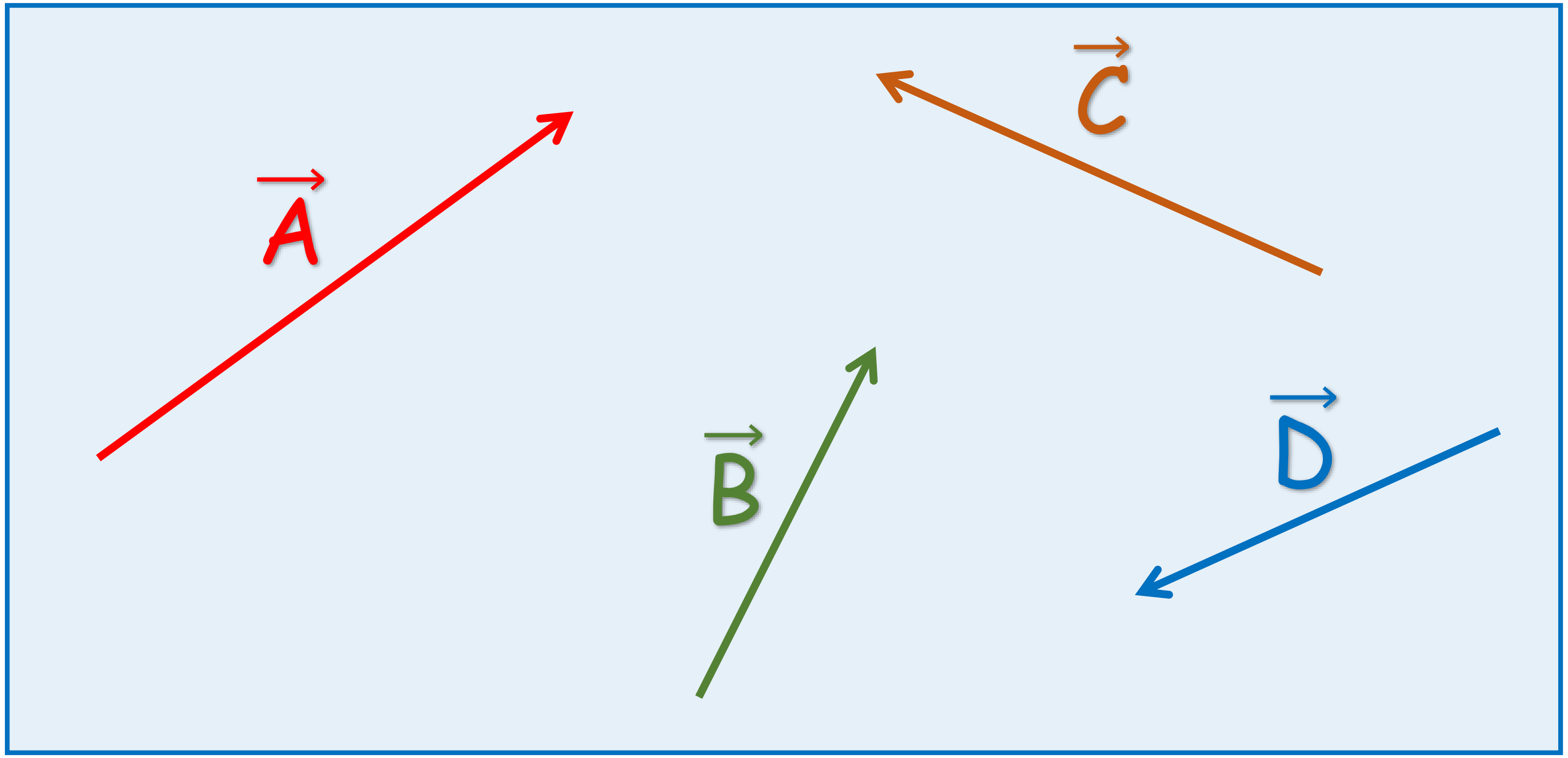
Co-terminus Vectors

Two or more vectors are said to be co-terminus vectors, if they have common terminal point.



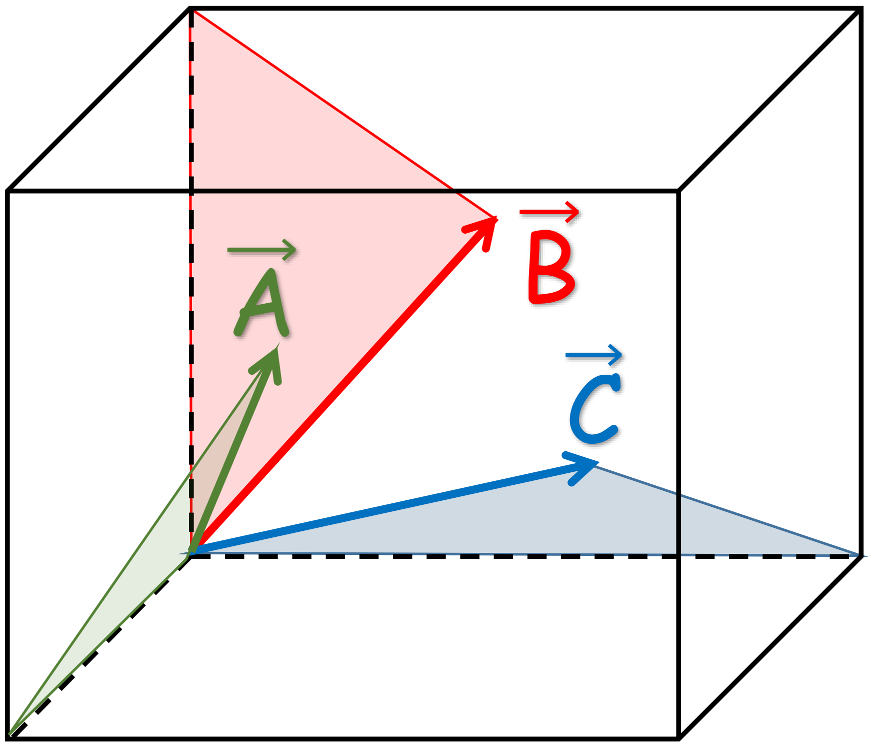
Coplanar Vectors

Three or more vectors are said to be coplanar vectors, if they lie in the same plane.



Non-coplanar Vectors

Three or more vectors are said to be non-coplanar vectors, if they are distributed in space.



Types of Vectors

(on the basis of effect)

Polar Vectors

Vectors having straight line effect are called polar vectors.

Examples:

- Displacement
- Velocity
- Acceleration
- Force

Axial Vectors

Vectors having rotational effect are called axial vectors.

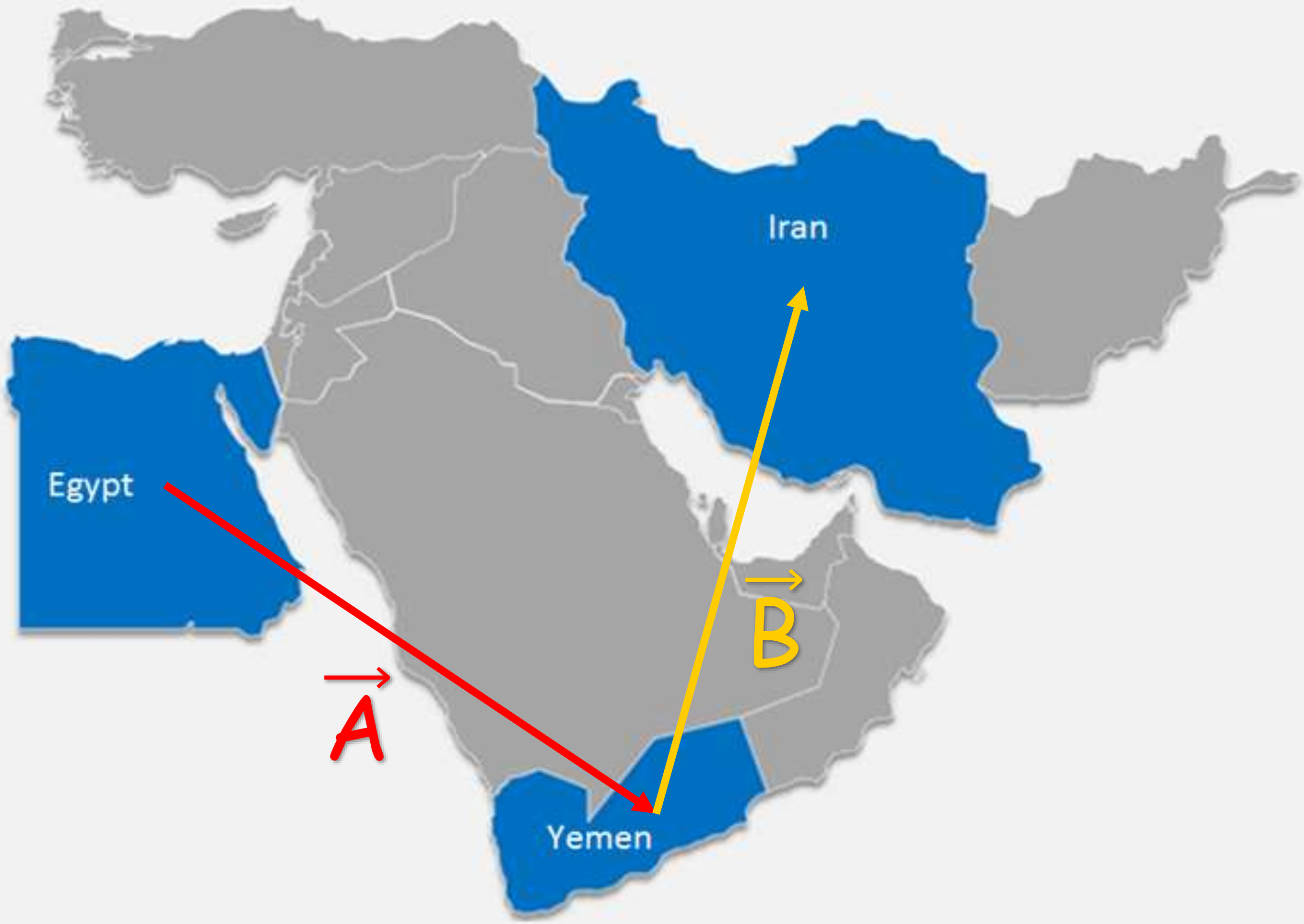
Examples:

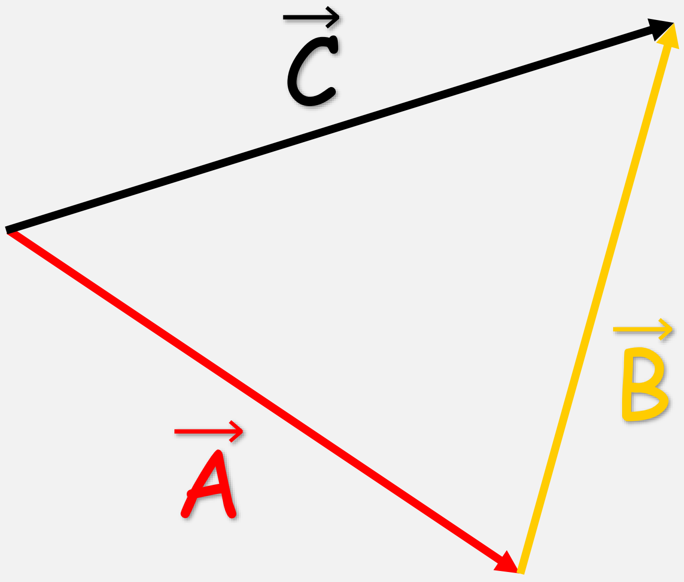
- Angular momentum
- Angular acceleration
- Angular velocity
- Torque

Vector Addition

(Geometrical Method)

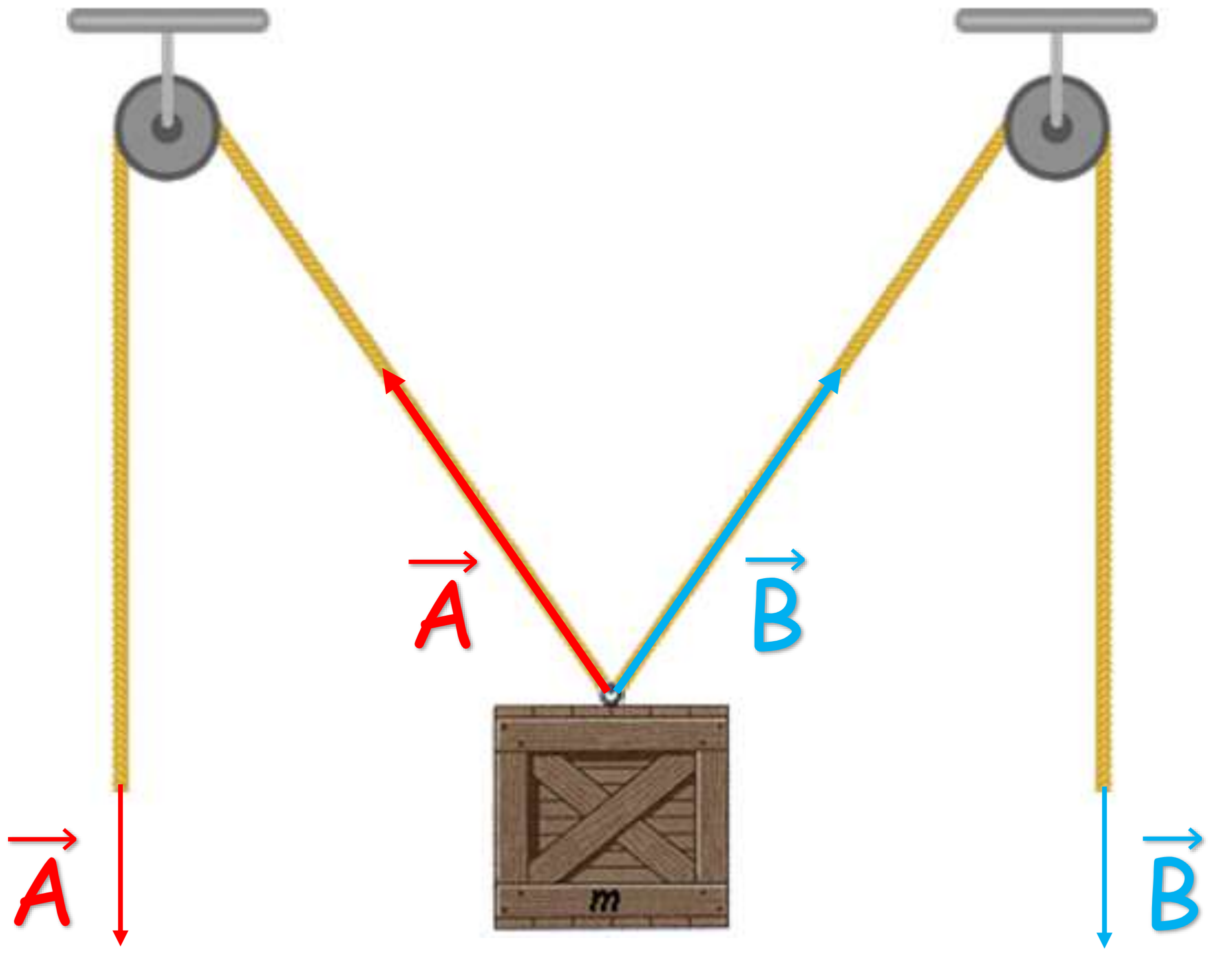
Triangle Law

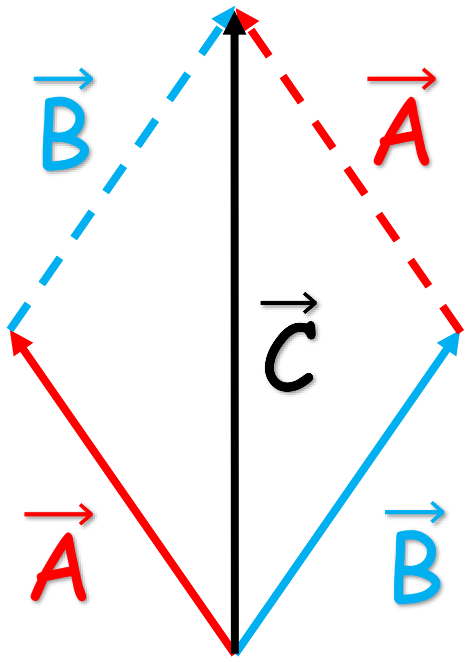




$$\vec{C} = \vec{A} + \vec{B}$$

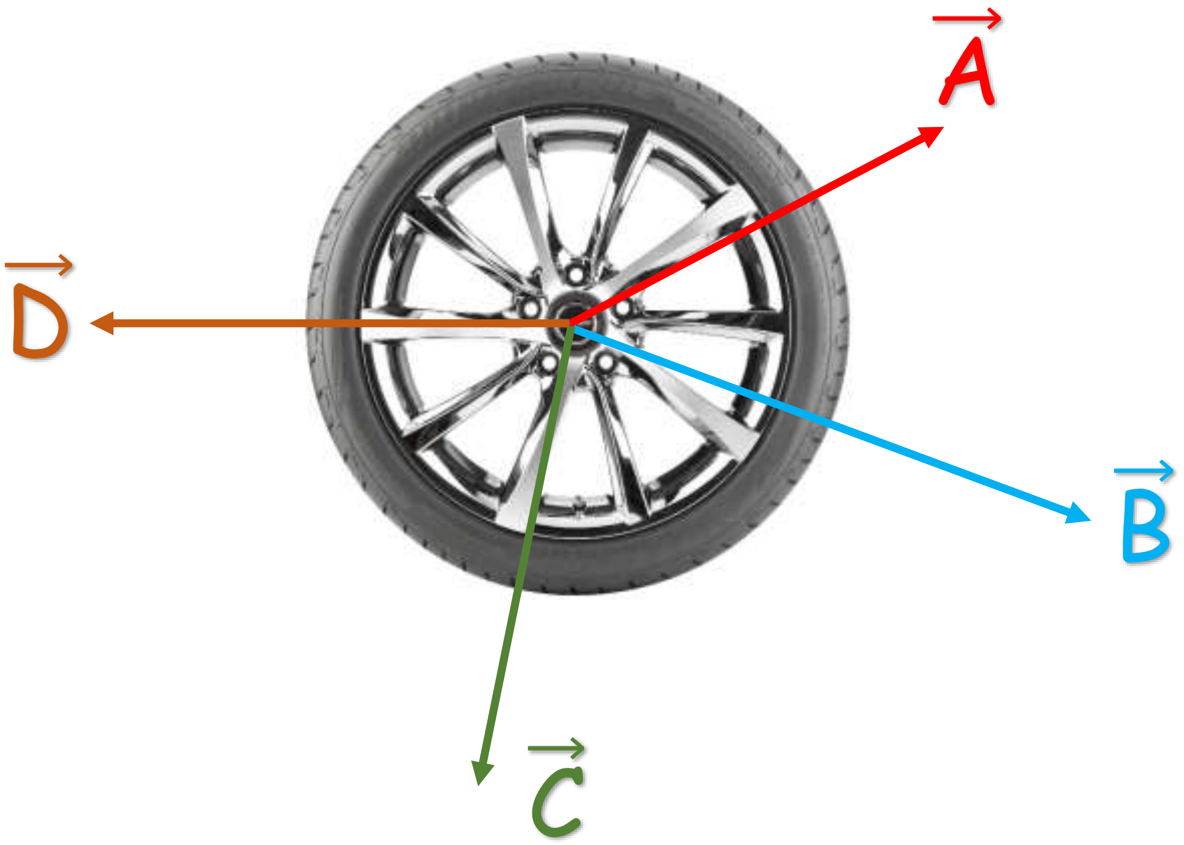
Parallelogram Law

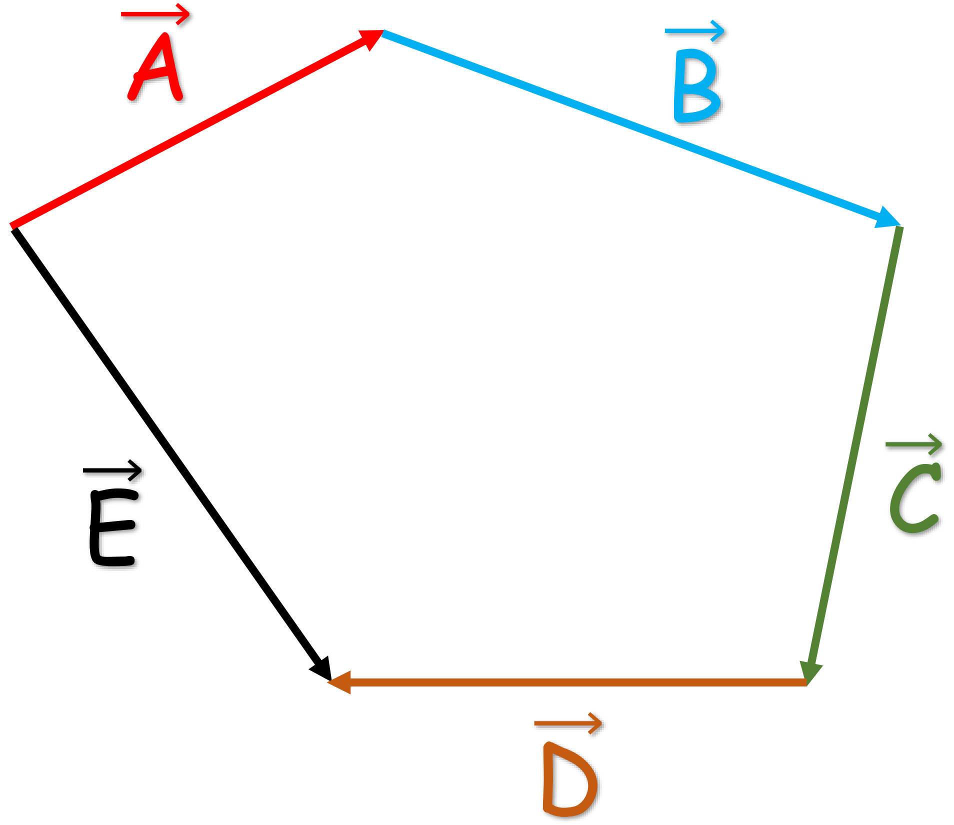




$$\vec{C} = \vec{A} + \vec{B}$$

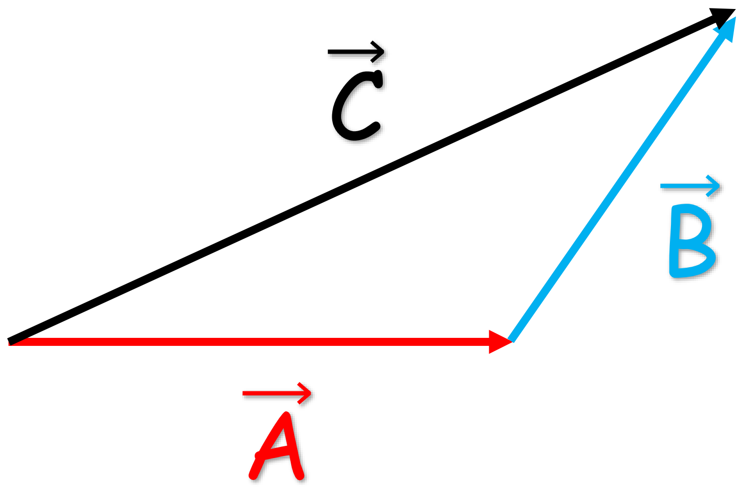
Polygon Law

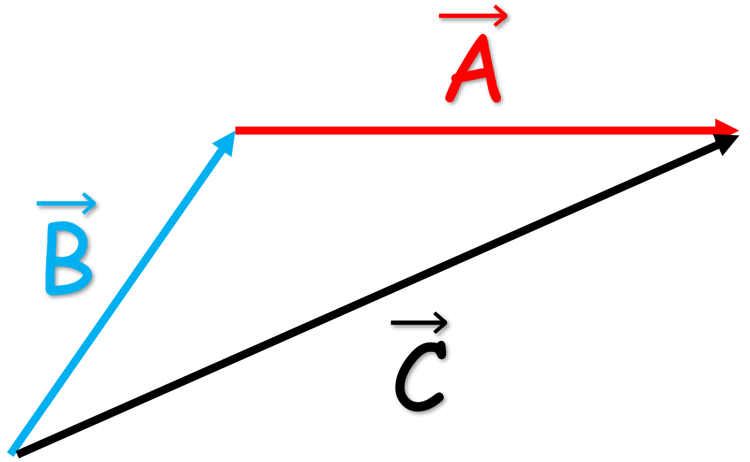




$$\vec{E} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

Commutative Property

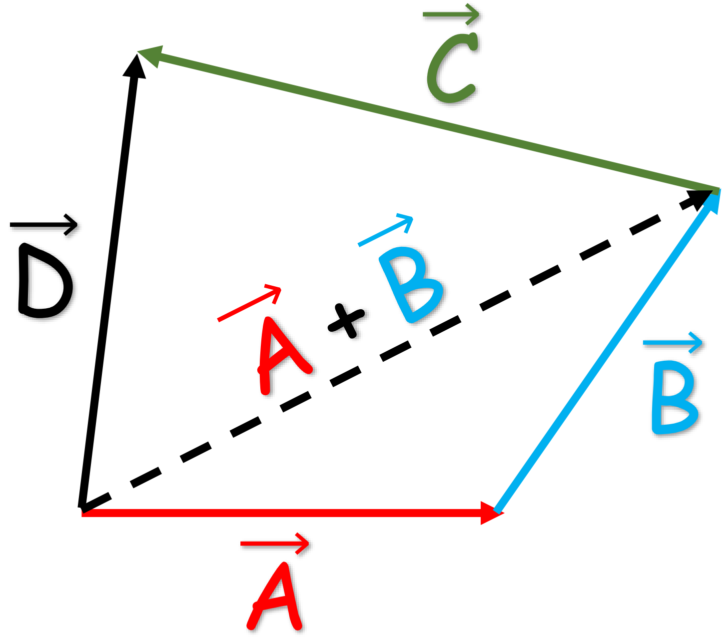


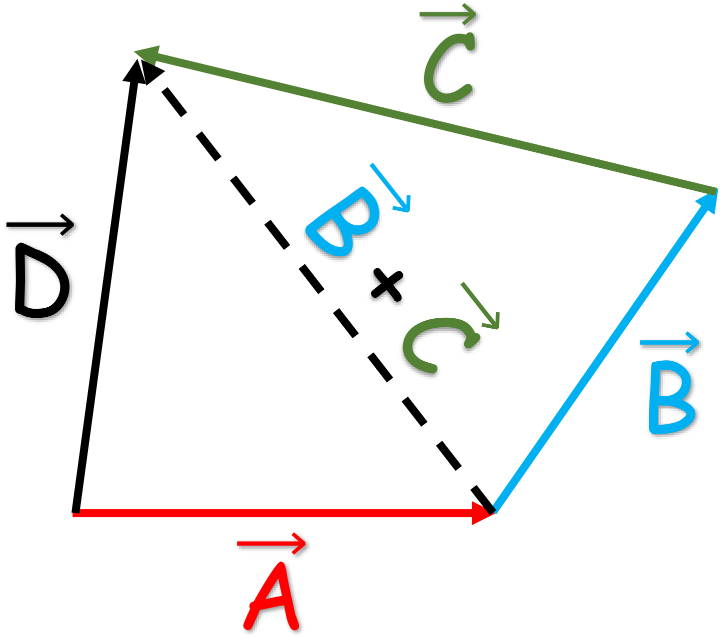


$$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Therefore, addition of vectors obey commutative law.

Associative Property

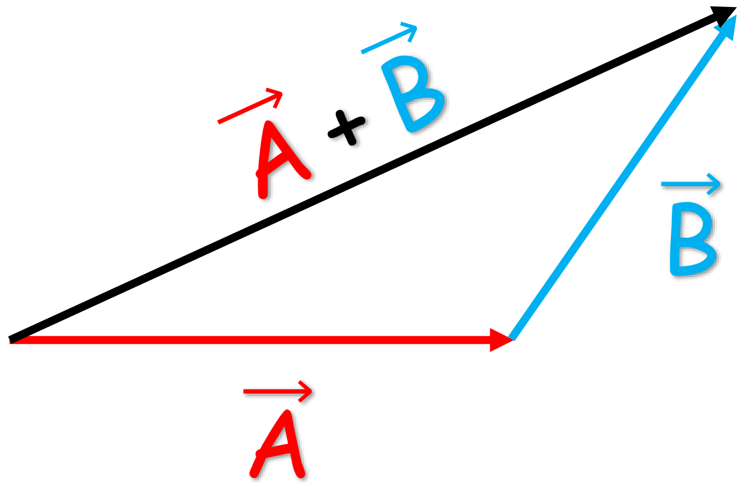


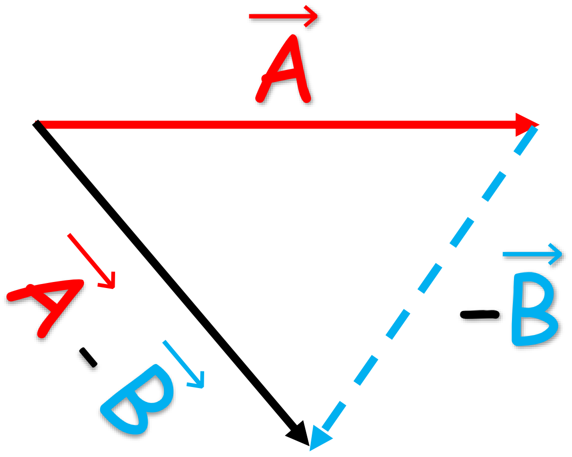


$$\vec{D} = (\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

Therefore, addition of vectors obey associative law.

Subtraction of vectors





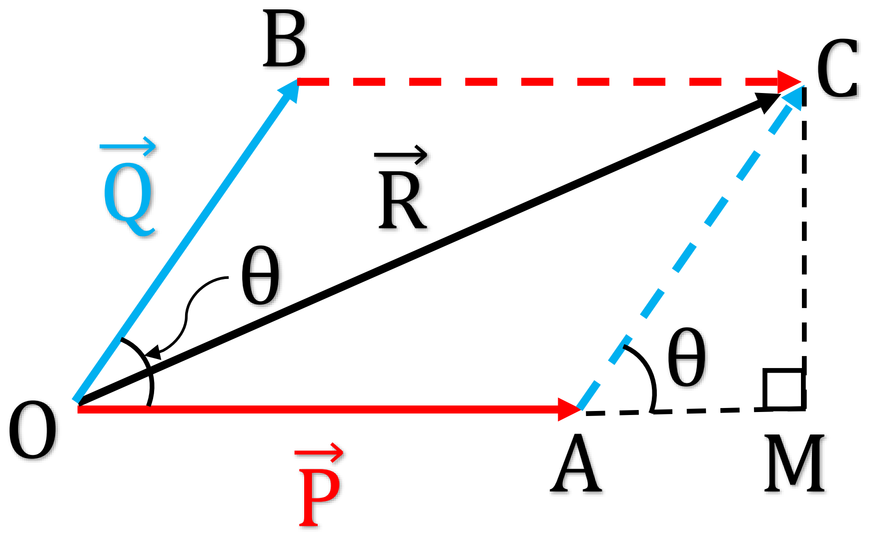
The subtraction of $\vec{B}$ from vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ to vector $\vec{A}$.

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Vector Addition

(Analytical Method)

Magnitude of Resultant



In ΔOCM,

$$OC^2 = OM^2 + CM^2$$

$$OC^2 = (OA + AM)^2 + CM^2$$

$$OC^2 = OA^2 + 2OA \times AM + AM^2 + CM^2$$

$$OC^2 = OA^2 + 2OA \times AM + AC^2$$

In ΔCAM,

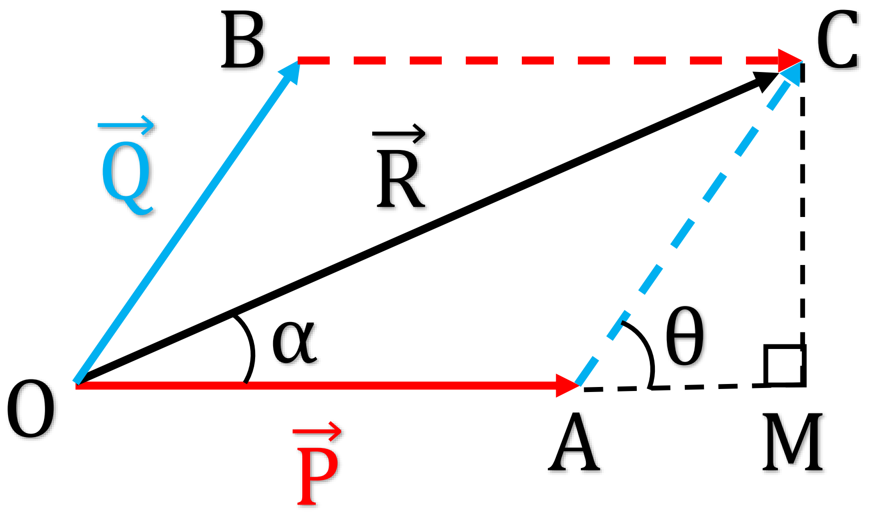
$$\cos\theta = \frac{AM}{AC} \quad \Rightarrow \quad AM = AC\cos\theta$$

$$OC^2 = OA^2 + 2OA \times AC\cos\theta + AC^2$$

$$R^2 = P^2 + 2P \times Q\cos\theta + Q^2$$

$$R = \sqrt{P^2 + 2PQ\cos\theta + Q^2}$$

Direction of Resultant



In ΔCAM,

$$\sin\theta = \frac{CM}{AC} \Rightarrow CM = AC\sin\theta$$

$$\cos\theta = \frac{AM}{AC} \Rightarrow AM = AC\cos\theta$$

In ΔOCM,

$$\tan\alpha = \frac{CM}{OM}$$

$$\tan\alpha = \frac{CM}{OA+AM}$$

$$\tan\alpha = \frac{AC\sin\theta}{OA+AC\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$$

Case I – Vectors are parallel ($\theta = 0°$)

$$\vec{P} + \vec{Q} = \vec{R}$$

Magnitude:

$$R = \sqrt{P^2 + 2PQ\cos 0° + Q^2}$$

$$R = \sqrt{P^2 + 2PQ + Q^2}$$

$$R = \sqrt{(P + Q)^2}$$

$$R = P + Q$$

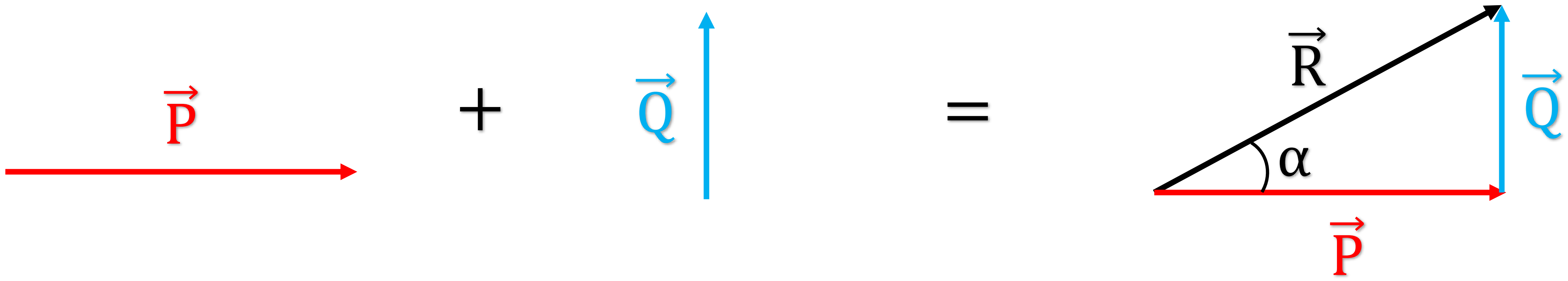
Direction:

$$\tan\alpha = \frac{Q\sin 0°}{P + Q\cos 0°}$$

$$\tan\alpha = \frac{0}{P + Q} = 0$$

$$\alpha = 0°$$

Case II – Vectors are perpendicular ($\theta = 90°$)



Magnitude:

$$R = \sqrt{P^2 + 2PQ\cos 90° + Q^2}$$

$$R = \sqrt{P^2 + 0 + Q^2}$$

$$R = \sqrt{P^2 + Q^2}$$

Direction:

$$\tan\alpha = \frac{Q\sin 90°}{P + Q\cos 90°} = \frac{Q}{P + 0}$$

$$\alpha = \tan^{-1}\left(\frac{Q}{P}\right)$$

Case III - Vectors are anti-parallel ($\theta = 180°$)

$$\vec{P} - \vec{Q} = \vec{R}$$

Magnitude:

$$R = \sqrt{P^2 + 2PQ\cos 180° + Q^2}$$

$$R = \sqrt{P^2 - 2PQ + Q^2}$$

$$R = \sqrt{(P - Q)^2}$$

$$R = P - Q$$

Direction:

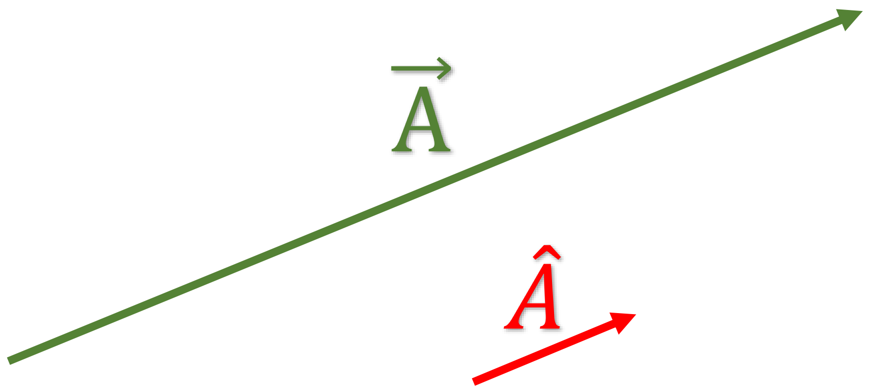
$$\tan\alpha = \frac{Q\sin 180°}{P + Q\cos 180°} = 0$$

If $P > Q$: $\alpha = 0°$

If $P < Q$: $\alpha = 180°$

Unit vectors

A unit vector is a vector that has a magnitude of exactly 1 and drawn in the direction of given vector.



- It lacks both dimension and unit.
- Its only purpose is to specify a direction in space.

Unit vectors

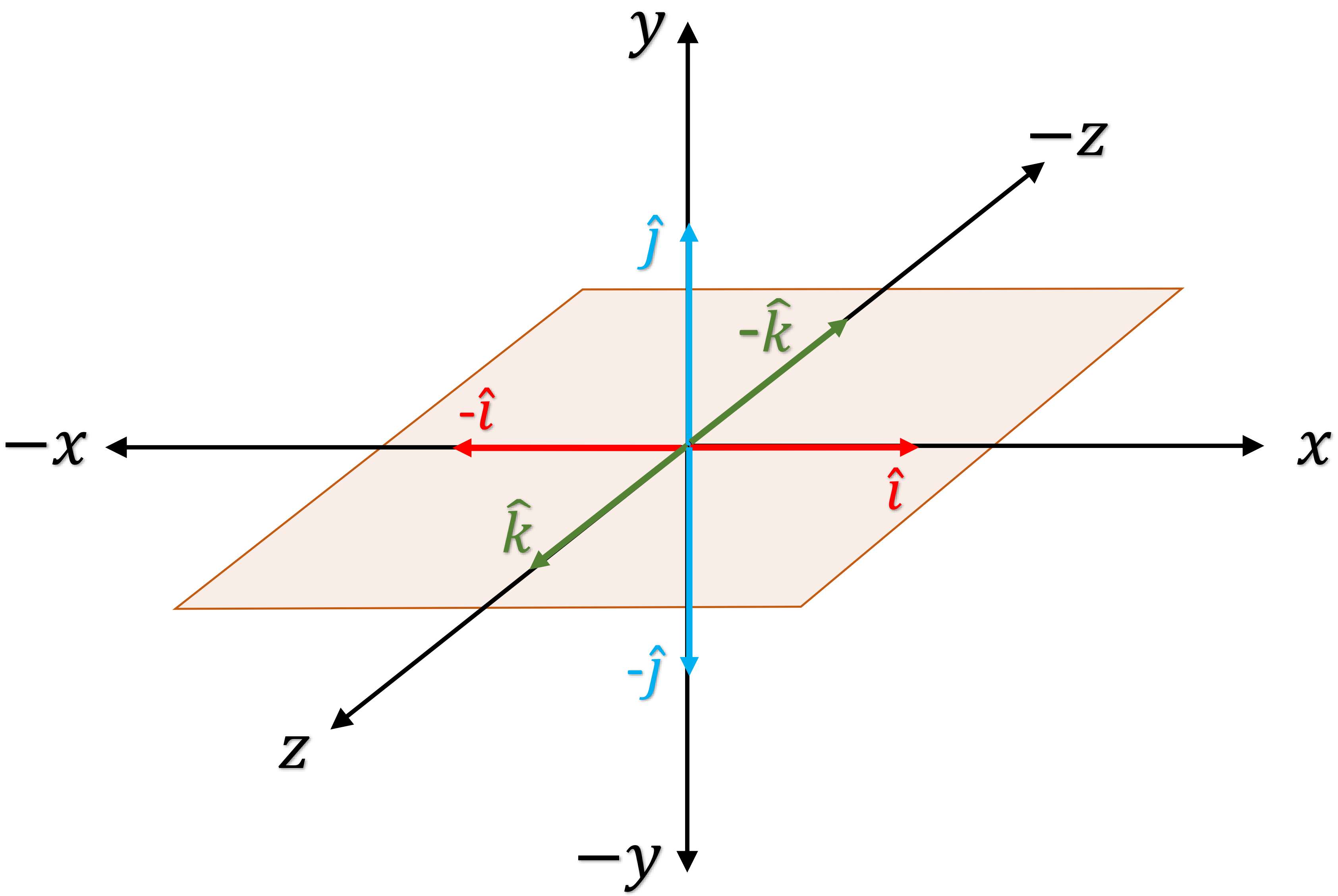
- A given vector can be expressed as a product of its magnitude and a unit vector.
- For example $\vec{A}$ may be represented as,

$$\vec{A} = A\,\hat{A}$$

A = magnitude of $\vec{A}$

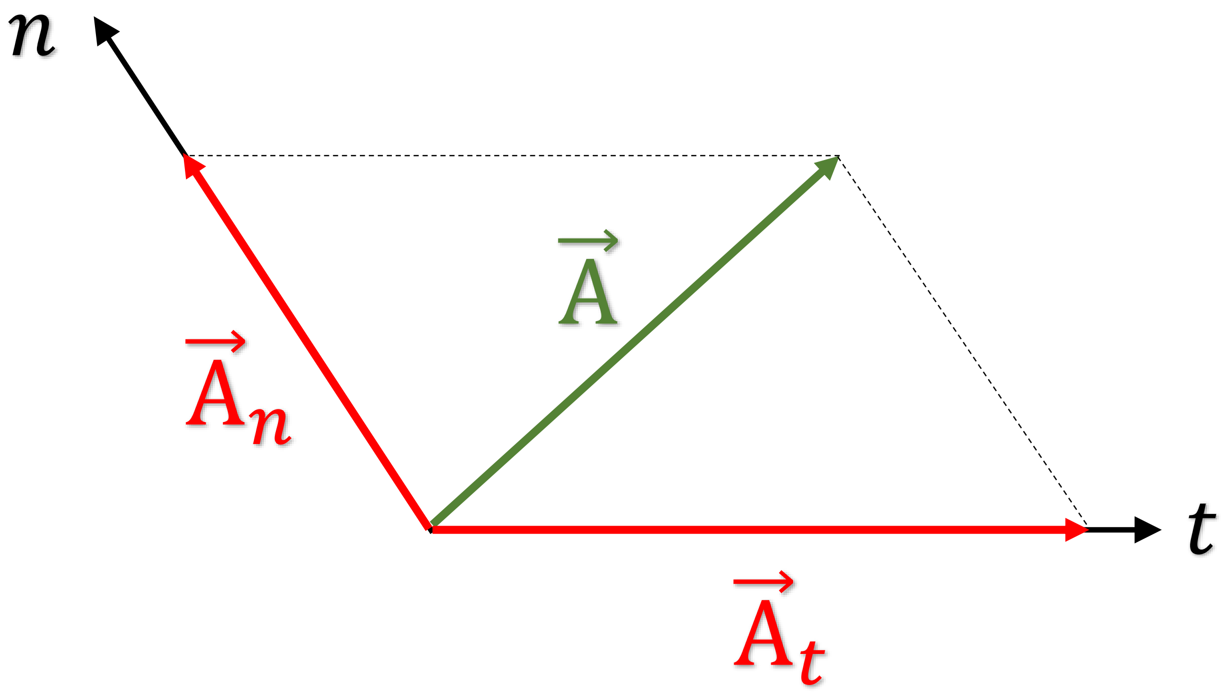
$\hat{A}$ = unit vector along $\vec{A}$

Cartesian unit vectors

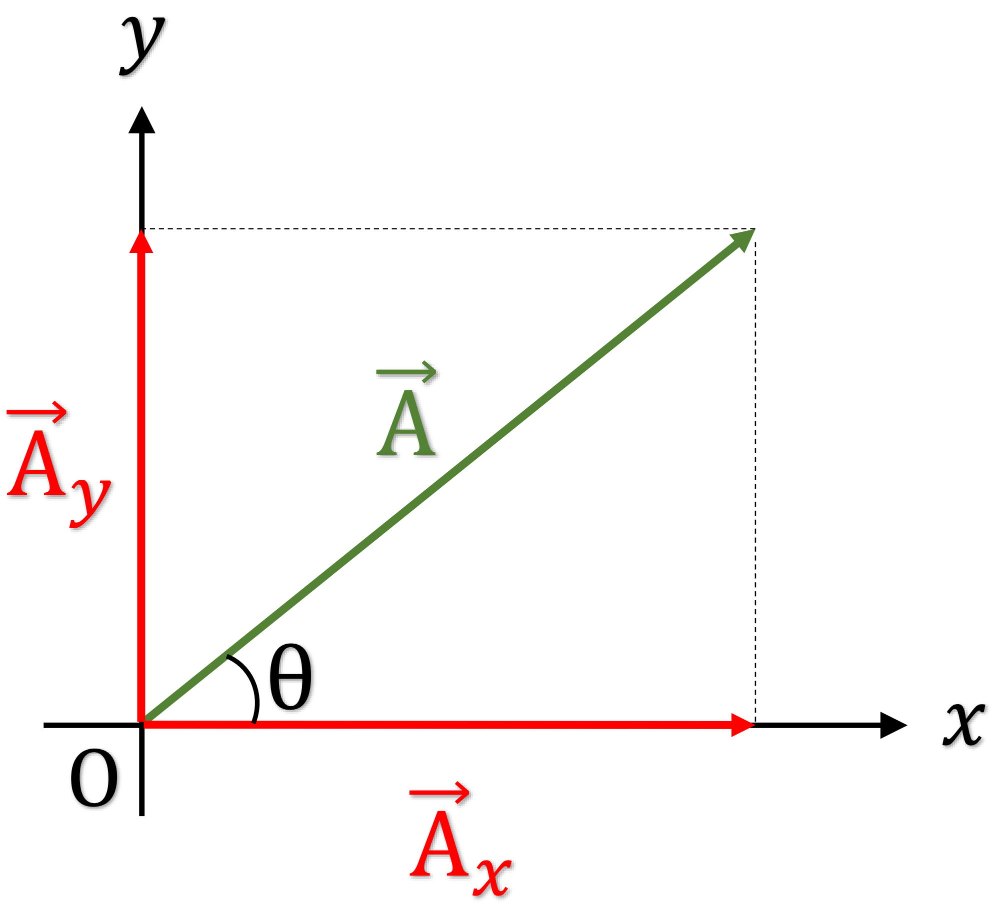


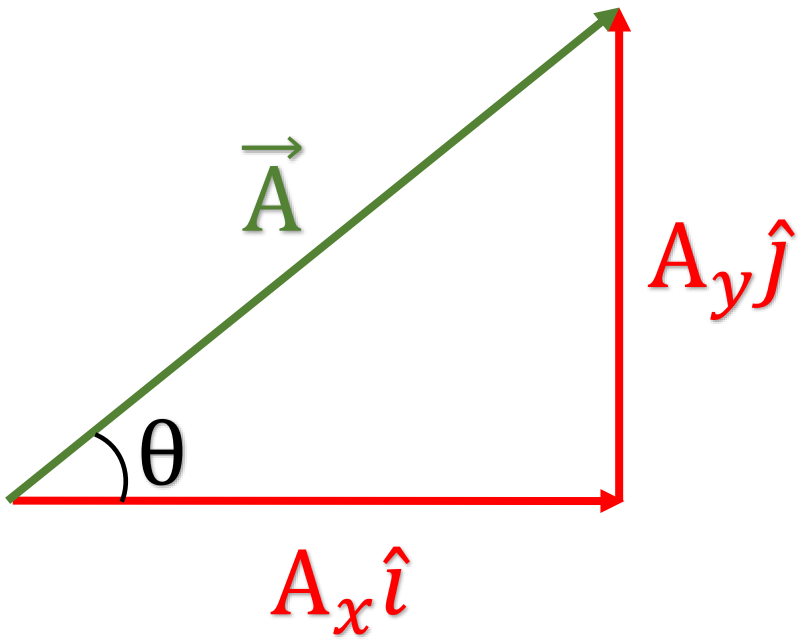
Resolution of a Vector

It is the process of splitting a vector into two or more vectors in such a way that their combined effect is same as that of the given vector.



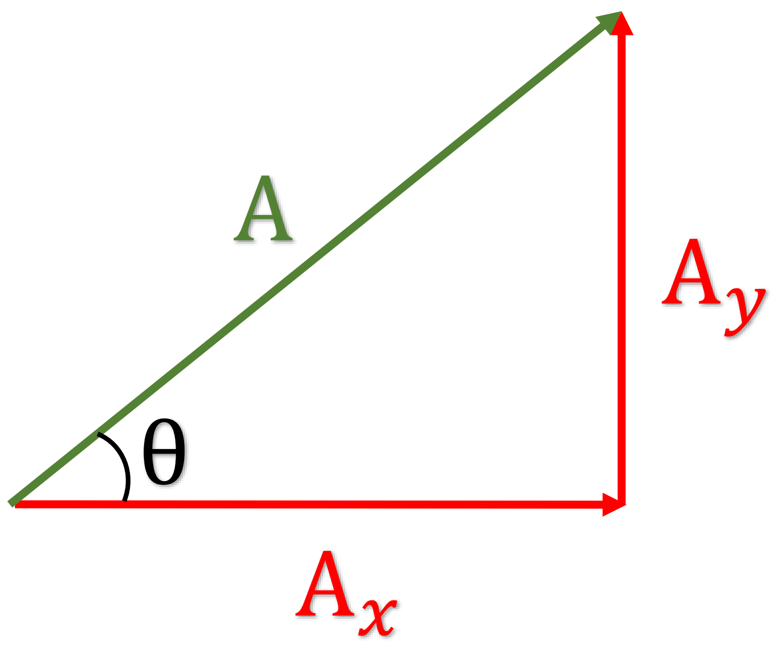
Rectangular Components of 2D Vectors





$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

Rectangular Components of 2D Vectors

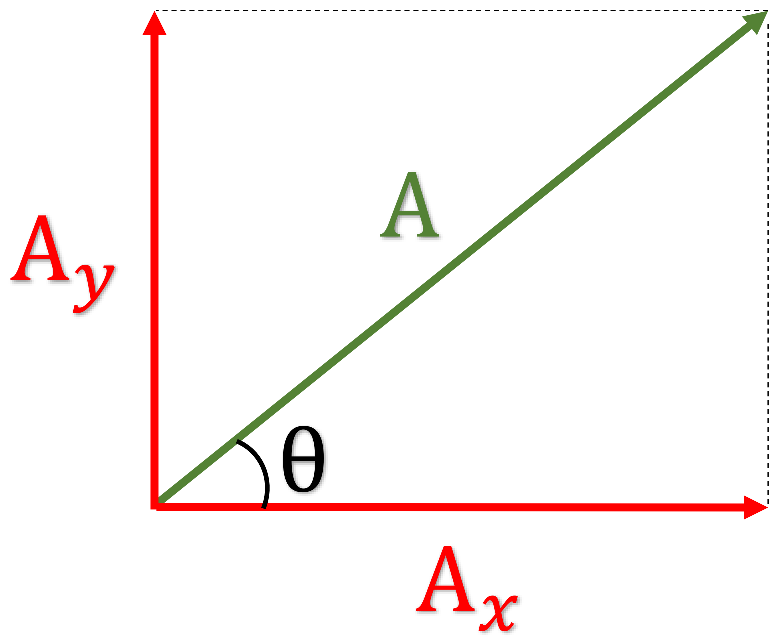


$$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta$$

$$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta$$

Magnitude & direction from components

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$



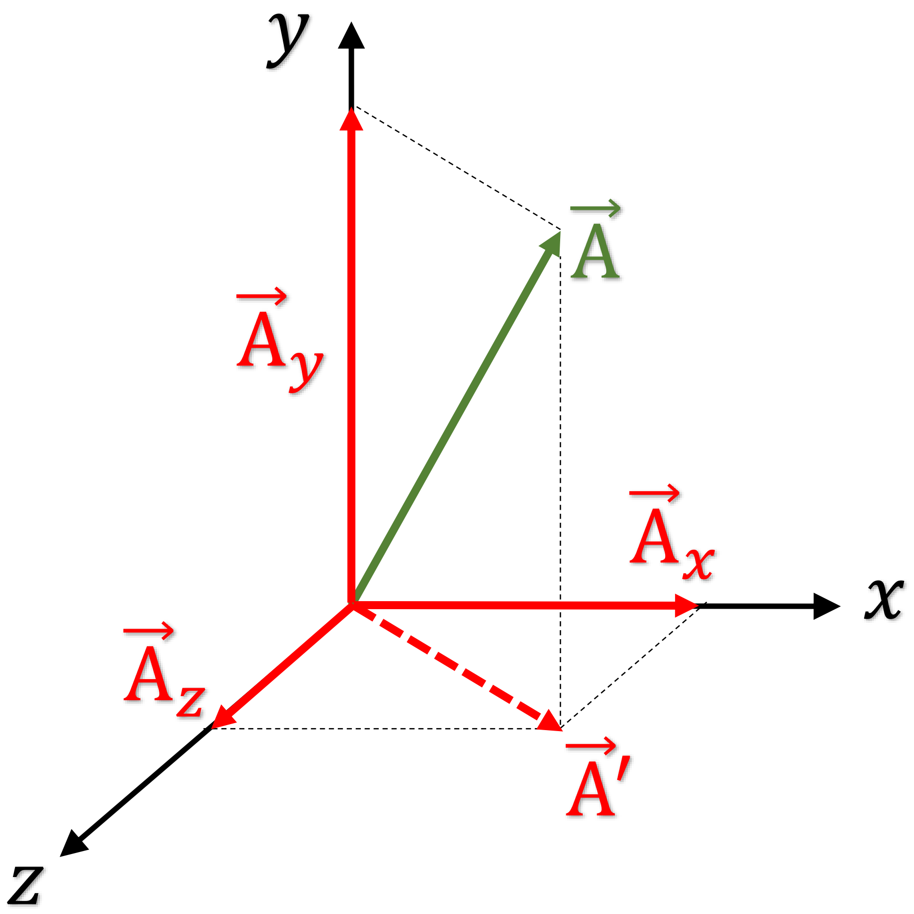
Magnitude:

$$A = \sqrt{A_x^2 + A_y^2}$$

Direction:

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Rectangular Components of 3D Vectors



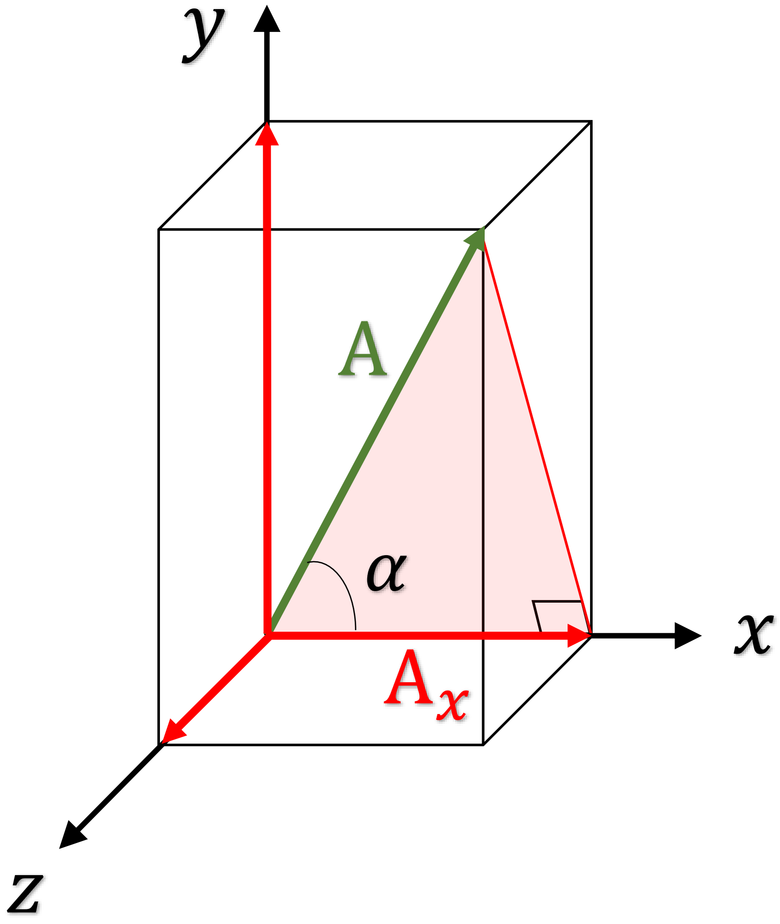
$$\vec{A} = \vec{A}' + \vec{A}_y$$

$$\vec{A} = \vec{A}_x + \vec{A}_z + \vec{A}_y$$

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

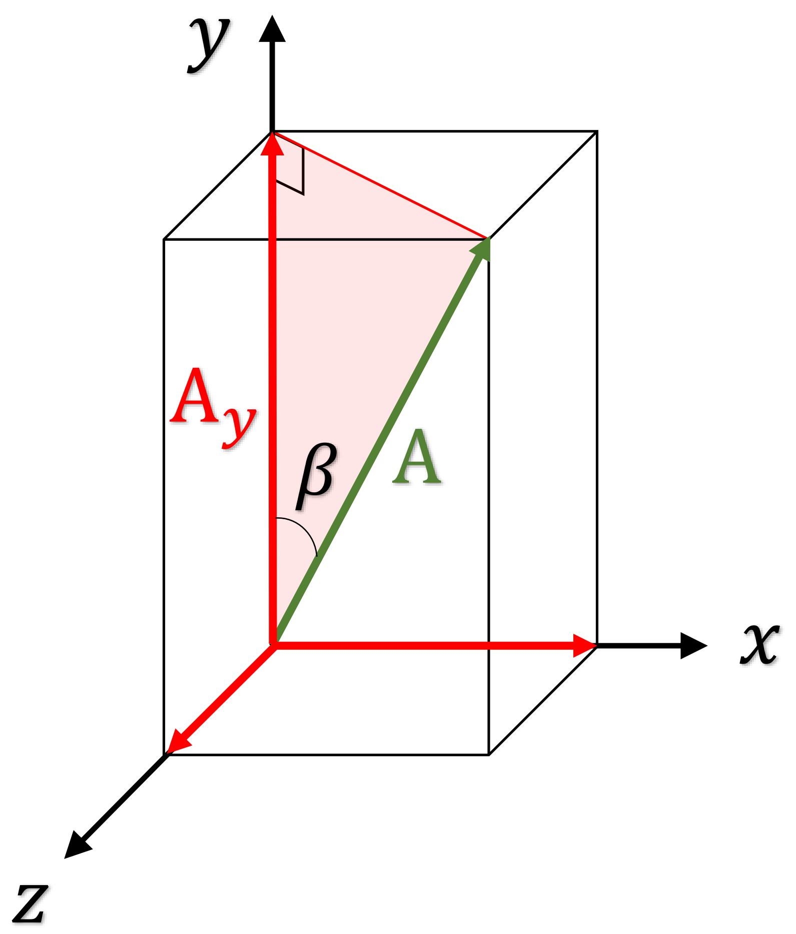
Rectangular Components of 3D Vectors



$$\cos\alpha = \frac{A_x}{A}$$

$$A_x = A\cos\alpha$$

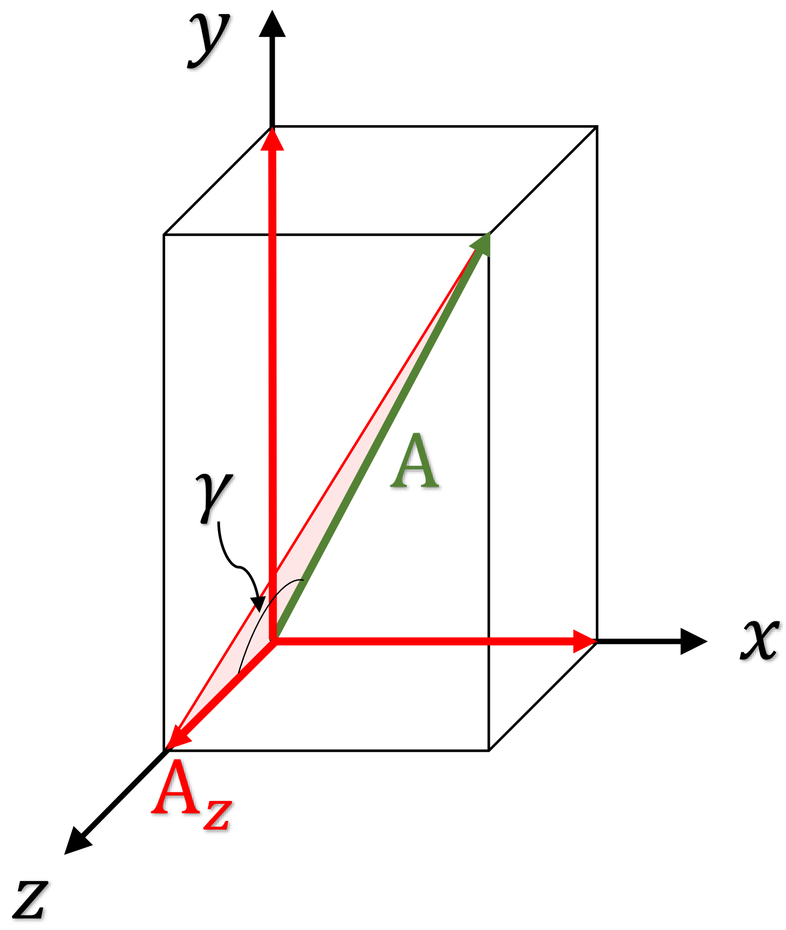
Rectangular Components of 3D Vectors



$$\cos\beta = \frac{A_y}{A}$$

$$A_y = A\cos\beta$$

Rectangular Components of 3D Vectors

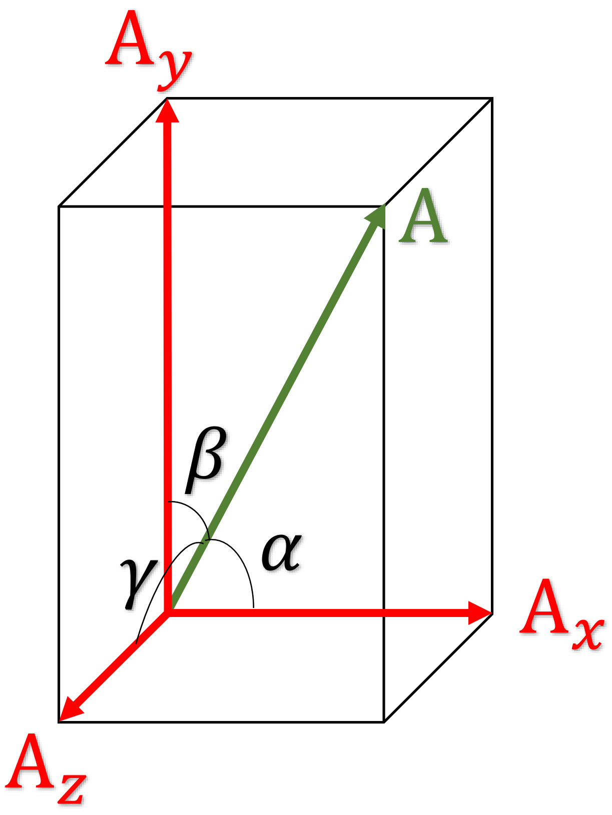


$$\cos\gamma = \frac{A_z}{A}$$

$$A_z = A\cos\gamma$$

Magnitude & direction from components

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$



Magnitude:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction:

$$\alpha = \cos^{-1}\left(\frac{A_x}{A}\right)$$

$$\beta = \cos^{-1}\left(\frac{A_y}{A}\right)$$

$$\gamma = \cos^{-1}\left(\frac{A_z}{A}\right)$$

Adding vectors by components

Let us have

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

then

$$\vec{R} = \vec{A} + \vec{B}$$

$$\vec{R} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} + B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

$$R_x\hat{i} + R_y\hat{j} + R_z\hat{k} = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} + (A_z+B_z)\hat{k}$$

$$R_x = (A_x + B_x)$$

$$R_y = (A_y + B_y)$$

$$R_z = (A_z + B_z)$$

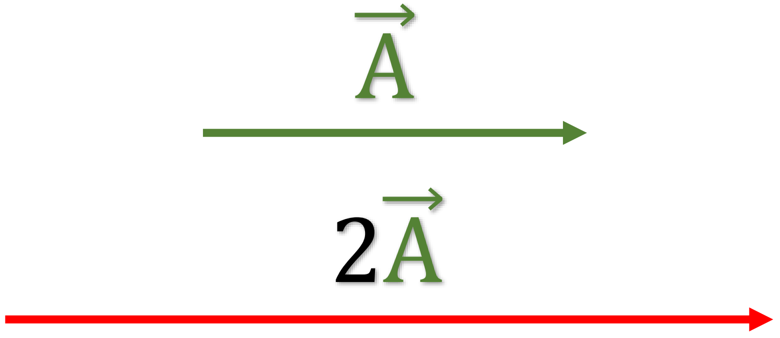
Multiplying vectors

Multiplying a vector by a scalar

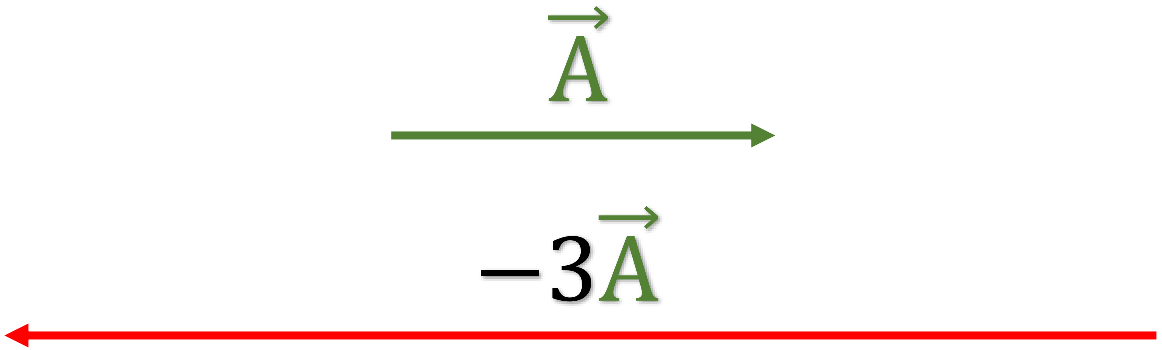
- If we multiply a vector $\vec{A}$ by a scalar s, we get a new vector.
- Its magnitude is the product of the magnitude of $\vec{A}$ and the absolute value of s.
- Its direction is the direction of $\vec{A}$ if s is positive but the opposite direction if s is negative.

Multiplying a vector by a scalar

If s is positive:



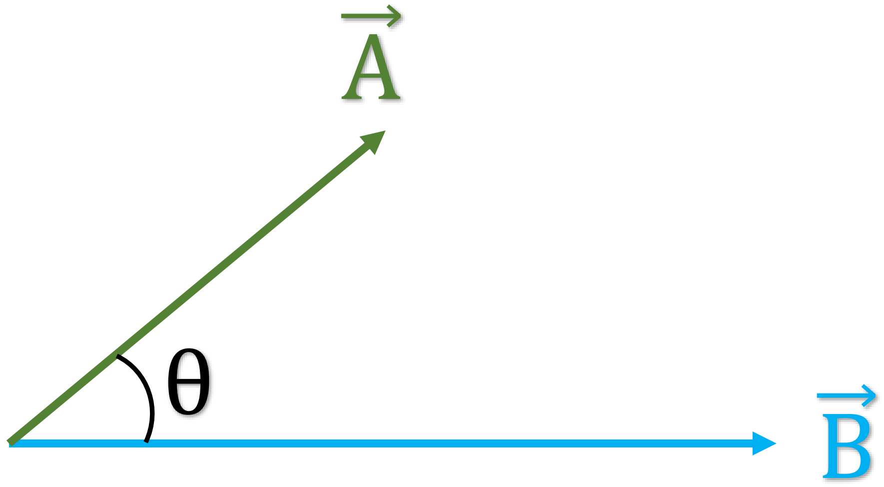
If s is negative:



Multiplying a vector by a vector

- There are two ways to multiply a vector by a vector:
- The first way produces a scalar quantity and called as scalar product (dot product).
- The second way produces a vector quantity and called as vector product (cross product).

Scalar product



$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Examples of scalar product

$$W = \vec{F} \cdot \vec{s}$$

$$W = Fs\cos\theta$$

W = work done

F = force

s = displacement

$$P = \vec{F} \cdot \vec{v}$$

$$P = Fv\cos\theta$$

P = power

F = force

v = velocity

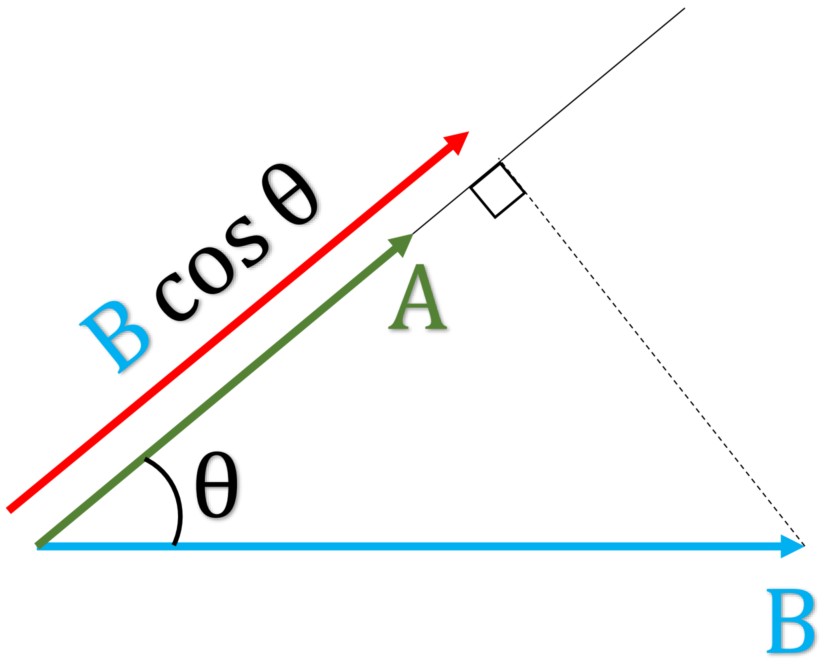
Geometrical meaning of Scalar dot product

A dot product can be regarded as the product of two quantities:

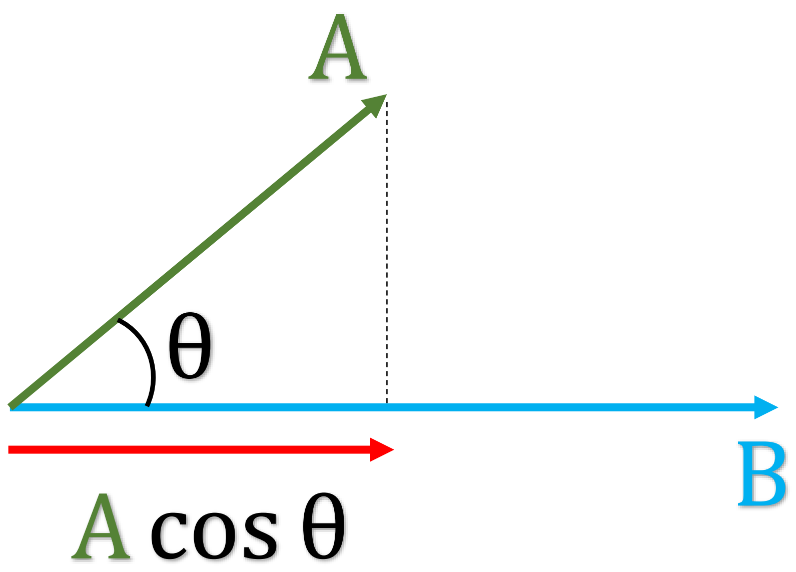
1. The magnitude of one of the vectors
2. The scalar component of the second vector along the direction of the first vector

Geometrical meaning of Scalar product

$$\vec{A} \cdot \vec{B} = A(B\cos\theta)$$



$$\vec{A} \cdot \vec{B} = (A\cos\theta)B$$



Properties of Scalar product

1

The scalar product is commutative.

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\vec{B} \cdot \vec{A} = BA\cos\theta$$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

Properties of Scalar product

2

The scalar product is distributive over addition.

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

Properties of Scalar product

3

The scalar product of two perpendicular vectors is zero.

$$\vec{A} \cdot \vec{B} = AB \cos 90\,°$$

$$\vec{A} \cdot \vec{B} = 0$$

Properties of Scalar product

4

The scalar product of two parallel vectors is maximum positive.

$$\vec{A} \cdot \vec{B} = AB\cos 0°$$

$$\vec{A} \cdot \vec{B} = AB$$

Properties of Scalar product

5

The scalar product of two anti-parallel vectors is maximum negative.

$$\vec{A} \cdot \vec{B} = AB \cos 180°$$

$$\vec{A} \cdot \vec{B} = -AB$$

Properties of Scalar product

6

The scalar product of a vector with itself is equal to the square of its magnitude.

$$\vec{A} \cdot \vec{A} = AA \cos 0\,°$$

$$\vec{A} \cdot \vec{A} = A^2$$

Properties of Scalar product

7

The scalar product of two same unit vectors is one and two different unit vectors is zero.

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = (1)(1)\cos 0° = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = (1)(1)\cos 90° = 0$$

Calculating scalar product using components

Let us have

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

then

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

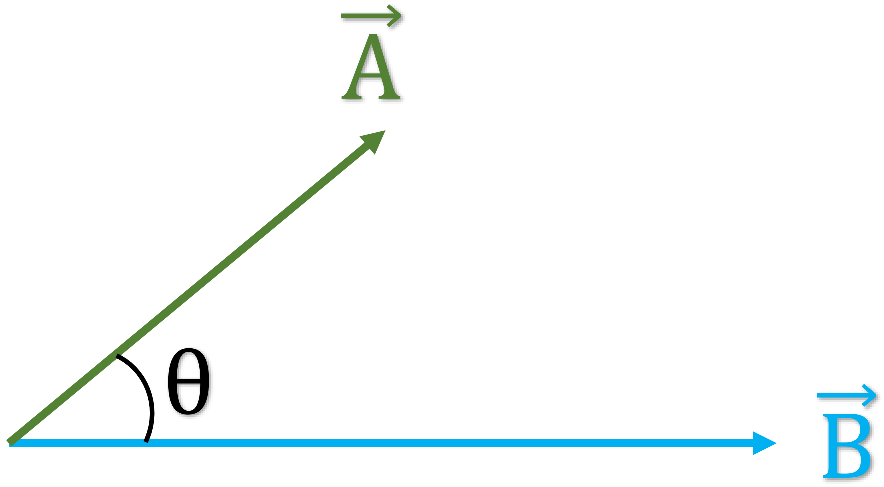
$$\vec{A}\cdot\vec{B} = A_x\hat{i}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_z\hat{k}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x\hat{i}\cdot\hat{i} + A_xB_y\hat{i}\cdot\hat{j} + A_xB_z\hat{i}\cdot\hat{k} + A_yB_x\hat{j}\cdot\hat{i} + A_yB_y\hat{j}\cdot\hat{j} + A_yB_z\hat{j}\cdot\hat{k} + A_zB_x\hat{k}\cdot\hat{i} + A_zB_y\hat{k}\cdot\hat{j} + A_zB_z\hat{k}\cdot\hat{k}$$

$$= A_xB_x(1) + A_xB_y(0) + A_xB_z(0) + A_yB_x(0) + A_yB_y(1) + A_yB_z(0) + A_zB_x(0) + A_zB_y(0) + A_zB_z(1)$$

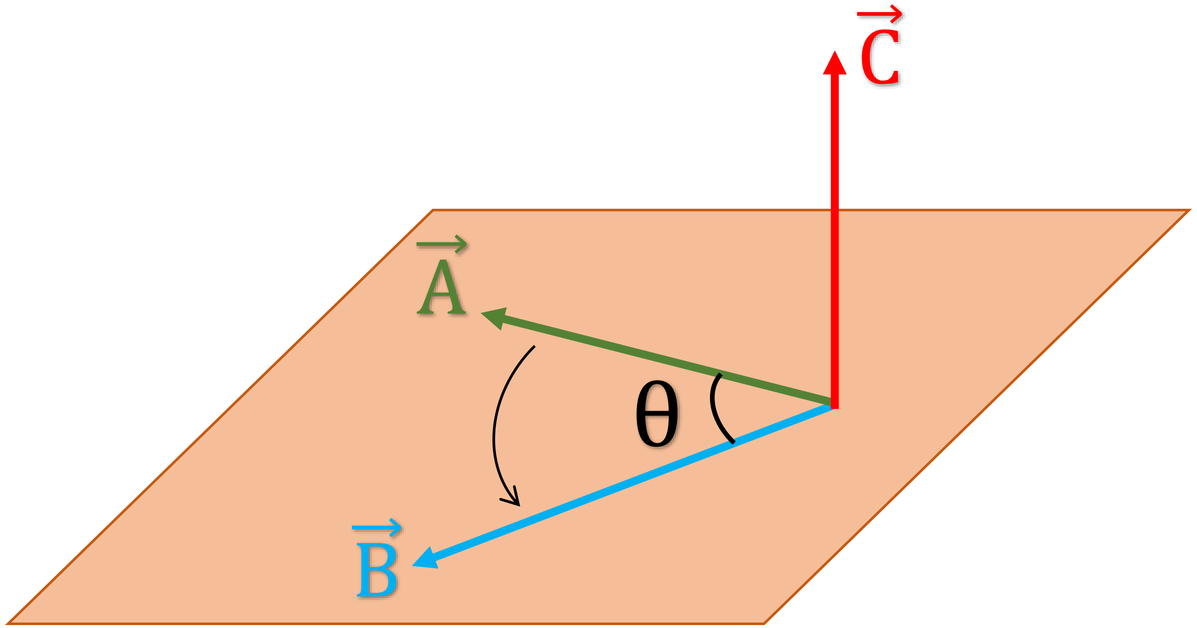
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Vector product



$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n} = \vec{C}$$

Right hand rule



Examples of vector product

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = rF \sin\theta \, \hat{n}$$

τ = torque

r = position

F = force

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = rp \sin\theta \, \hat{n}$$

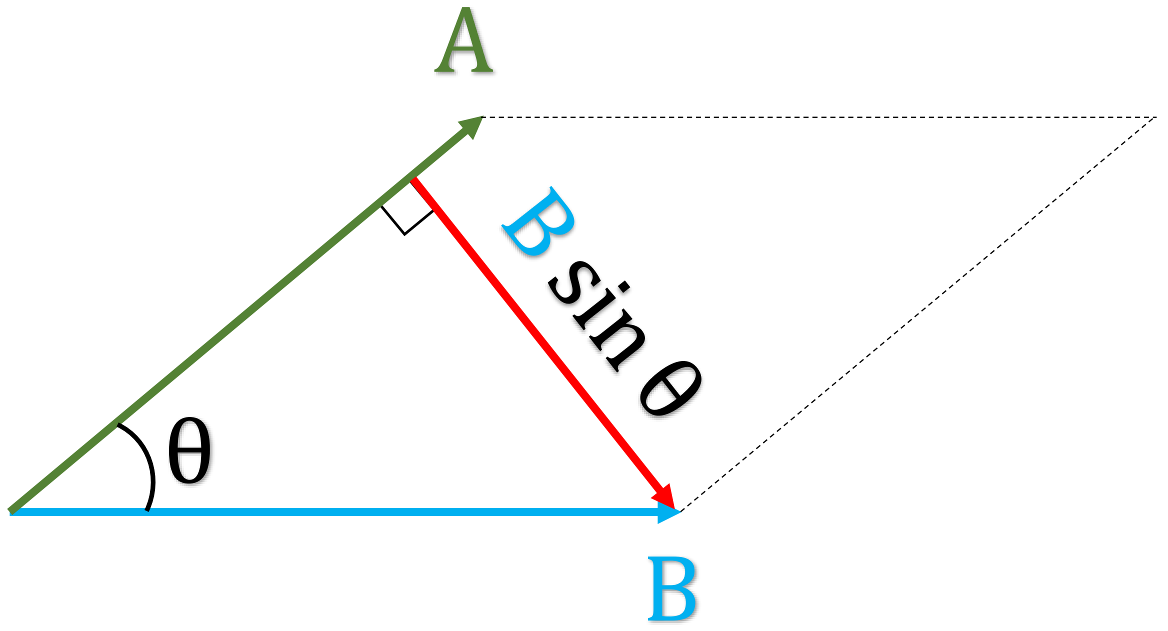
L = angular momentum

r = position

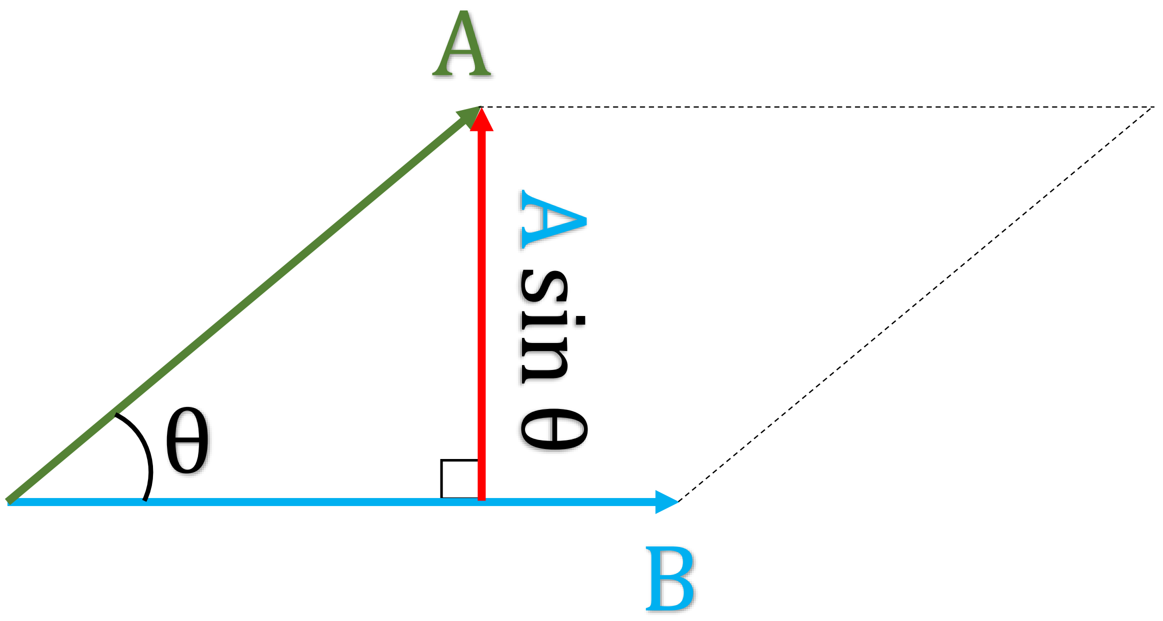
p = linear momentum

Geometrical meaning of Vector product

$$|\vec{A} \times \vec{B}| = A(B \sin \theta)$$



$$|\vec{A} \times \vec{B}| = (A \sin \theta)B$$



$|\vec{A} \times \vec{B}|$ = Area of parallelogram made by two vectors

Properties of Vector product

1

The vector product is anti-commutative.

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$

$$\vec{B} \times \vec{A} = BA \sin\theta \, (-\hat{n}) = -AB \sin\theta \, \hat{n}$$

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

Properties of Vector product

2

The vector product is distributive over addition.

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

Properties of Vector product

3

The magnitude of the vector product of two perpendicular vectors is maximum.

$$|\vec{A} \times \vec{B}| = AB \sin 90°$$

$$|\vec{A} \times \vec{B}| = AB$$

Properties of Vector product

4

The vector product of two parallel vectors is a null vector.

$$\vec{A} \times \vec{B} = AB \sin 0° \, \hat{n}$$

$$\vec{A} \times \vec{B} = \vec{0}$$

Properties of Vector product

5

The vector product of two anti-parallel vectors is a null vector.

$$\vec{A} \times \vec{B} = AB \sin 180° \, \hat{n}$$

$$\vec{A} \times \vec{B} = \vec{0}$$

Properties of Vector product

6

The vector product of a vector with itself is a null vector.

$$\vec{A} \times \vec{A} = AA \sin 0° \, \hat{n}$$

$$\vec{A} \times \vec{A} = \vec{0}$$

Properties of Vector product

7

The vector product of two same unit vectors is a null vector.

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k}$$

$$= (1)(1)\sin 0^{\circ}\hat{n} = \vec{0}$$

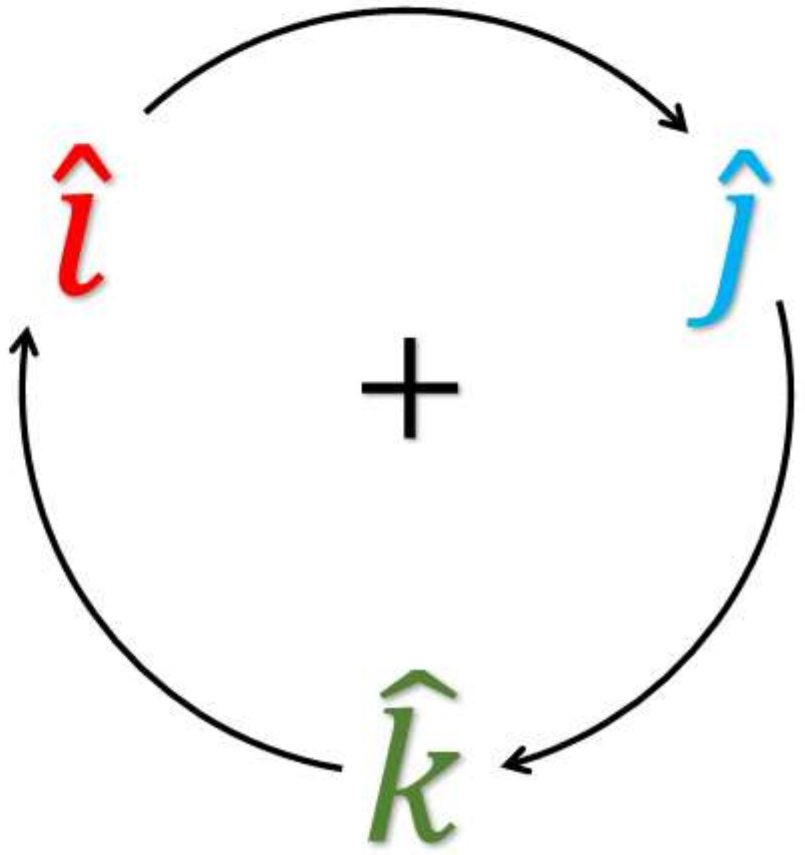
Properties of Vector product

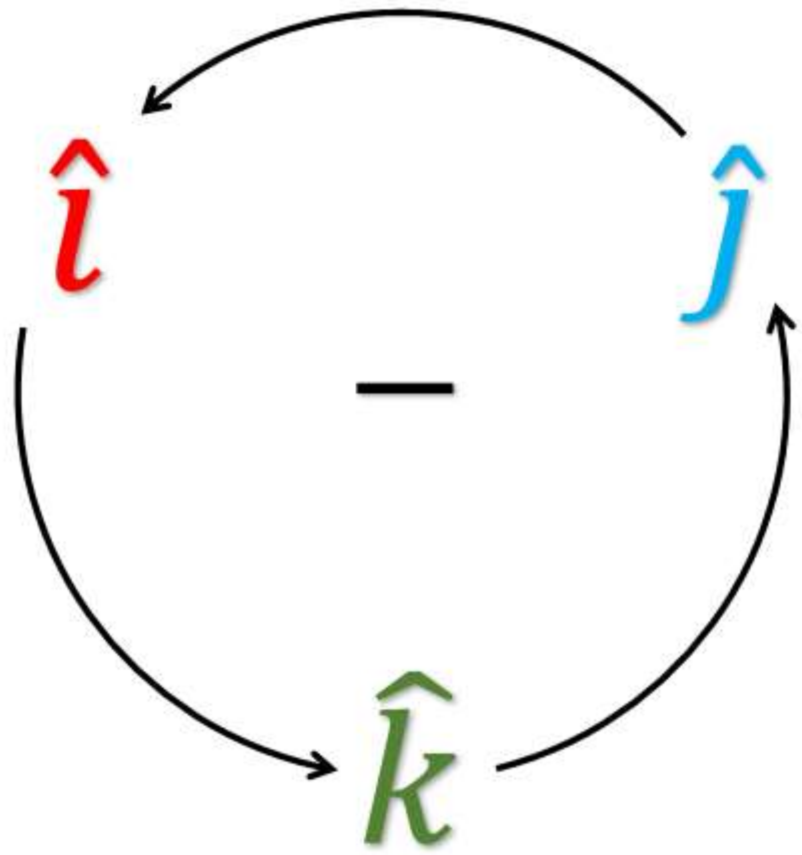
8

The vector product of two different unit vectors is a third unit vector.

$\hat{\imath} \times \hat{\jmath} = \hat{k}$	$\hat{\jmath} \times \hat{\imath} = -\hat{k}$
$\hat{\jmath} \times \hat{k} = \hat{\imath}$	$\hat{k} \times \hat{\jmath} = -\hat{\imath}$
$\hat{k} \times \hat{\imath} = \hat{\jmath}$	$\hat{\imath} \times \hat{k} = -\hat{\jmath}$

Aid to memory





Calculating vector product using components

Let us have

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

then

$$\vec{A} \times \vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right) \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right)$$

$$\vec{A} \times \vec{B} = A_x\hat{\imath} \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right) + A_y\hat{\jmath} \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right) + A_z\hat{k} \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right)$$

$$= A_xB_x\hat{\imath} \times \hat{\imath} + A_xB_y\hat{\imath} \times \hat{\jmath} + A_xB_z\hat{\imath} \times \hat{k} + A_yB_x\hat{\jmath} \times \hat{\imath} + A_yB_y\hat{\jmath} \times \hat{\jmath} + A_yB_z\hat{\jmath} \times \hat{k} + A_zB_x\hat{k} \times \hat{\imath} + A_zB_y\hat{k} \times \hat{\jmath} + A_zB_z\hat{k} \times \hat{k}$$

$$= A_xB_x(\vec{0}) + A_xB_y(\hat{k}) + A_xB_z(-\hat{\jmath}) + A_yB_x(-\hat{k}) + A_yB_y(\vec{0}) + A_yB_z(\hat{\imath}) + A_zB_x(\hat{\jmath}) + A_zB_y(-\hat{\imath}) + A_zB_z(\vec{0})$$

$$= A_yB_z(\hat{\imath}) - A_zB_y(\hat{\imath}) + A_zB_x(\hat{\jmath}) - A_xB_z(\hat{\jmath}) + A_xB_y(\hat{k}) - A_yB_x(\hat{k})$$

Calculating vector product using components

$$\vec{A} \times \vec{B} = \hat{\imath}\,(A_yB_z - A_zB_y) - \hat{\jmath}\,(A_xB_z - A_zB_x) + \hat{k}\,(A_xB_y - A_yB_x)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Thank you