

Introduction

The first linear programming formulation of a problem that is equivalent to the general linear programming problem was given by Leonid Kantorovich in 1939, who also proposed a method for solving it. He developed it during World War II as a way to plan expenditures and returns so as to reduce costs to the army and increase losses incurred by the enemy. About the same time as Kantorovich, the Dutch-American economist T. C. Koopmans formulated classical economic problems as linear programs. Kantorovich and Koopmans later shared the 1975 Nobel prize in economics.[1] In 1941, Frank Lauren Hitchcock also formulated transportation problems as linear programs and gave a solution very similar to the later Simplex method;[2] Hitchcock had died in 1957 and the Nobel prize is not awarded posthumously. In 1947, George B. Dantzig published the simplex method and John von Neumann developed the theory of duality as a linear optimization solution, and applied it in the field of game theory. Postwar, many industries found its use in their daily planning. Dantzig's original example was to find the best assignment of 70 people to 70 jobs. The computing power required to test all the permutations to select the best assignment is vast; the number of possible configurations exceeds the number of particles in the observable universe. However, it takes only a moment to find the optimum solution by posing the problem as a linear program and applying the simplex algorithm. The theory behind linear programming drastically reduces the number of possible solutions that must be checked.

The linear-programming problem was first shown to be solvable in polynomial time by Leonid Khachiyan in 1979, but a larger theoretical and practical breakthrough in the field came in 1984 when Narendra Karmarkar introduced a new interior-point method for solving linear-programming problems the linear programming problem consists of the following three parts:

A linear function to be maximized

$Z = c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n$ subject to
 problem constraints of the following form

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n \leq b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3n}x_n \leq b_3$$

$$\dots$$

$$\dots$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n \leq b_m \text{ where}$$

Non-negative variables $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, \dots, x_n \geq 0$ and z is a objective function.

linear programming is the process of taking various linear inequalities relating to some situation, and finding the "best" obtainable under those conditions. In "real-life", linear programming is part of a very important area of mathematics called "optimization techniques". This field of study are used every day in the organization and allocation of resources. The general process for solving linear-programming exercises is to graph the inequalities (called the "constraints") to form a walled-off area on the x,y -plane (called the "feasibility region"). Then you figure out the coordinates of the corners of this feasibility region (that is, you find the intersection points of the various pairs of lines), and test these corner points in the formula (called the "optimization equation") for which you're trying to find the highest or lowest value.

TD formulate lpp problem:-

consider the daily requirement of vitamins v_1 & v_2 and mineral M for a certain person is at least 30 units of v_1 , 60 units of v_2 but not more than 40 units of M. He meets this requirement by taking two brands of tablets A and B

Tablet A has 3 units of v_1 , 4 units of v_2 and 1 unit of M

Tablet B has 1 units of v_1 , 3 units of v_2 and 21 unit of M

Tablet A costs Rs.2 and costs Re.1 determine the quantities of A & B he should take to minimise his expenditure

Suppose x units of A and y units of B

Therefore we have to minimise his expenditure $Z=2x+y$

	A(x)	B(y)	Requirements
v_1	3	1	30
v_2	4	3	60
M	1	2	40
	Rs.2	Re.1	

Therefore linear conditions are $3x+y \geq 30, 4x+3y \geq 60, x+2y \leq 40$

Find the maximum and minimum value of $z = 3x + 4y$ subject to the following constraints:

$x+2y \leq 2, 3x-y \geq 0, x-y \leq 2$

The three inequalities are the constraints. The shaded region is the feasibility region. The equation $z = 3x + 4y$ is the optimization equation. Here it is to find the (x, y) corner points of the feasibility region that return the largest and smallest values of z.

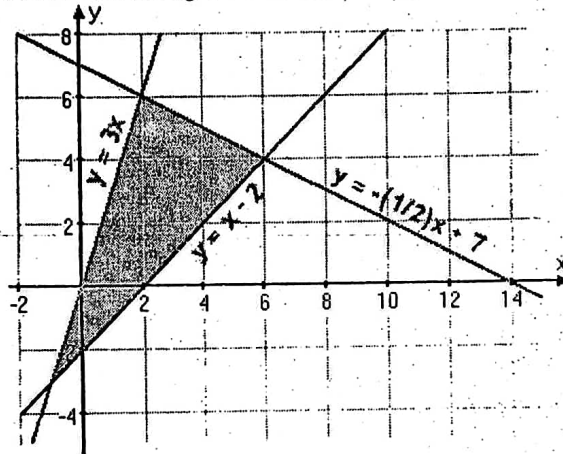
First step is to solve each inequality for the more-easily graphed equivalent forms:

$x+2y \leq 2 \Rightarrow y \leq -1/2x + 1$

$3x-y \geq 0 \Rightarrow y \leq 3x$

$x-y \leq 2 \Rightarrow y \geq x-2$

The graph of system is as follows :2006-2011 All Rights Reserved



To find the corner points which are not clear from the graph, consider pair the lines of system of linear equations and solve:

$y = -(1/2)x + 7$ $y = x - 2$	$y = -(1/2)x + 7$ $y = 3x$	$y = 3x$ $y = x - 2$
$-(1/2)x + 7 = x - 2$ $-x + 14 = 2x - 4$ $18 = 3x$ $6 = x$ $y = (6) - 2 = 4$	$-(1/2)x + 7 = 3x$ $-x + 14 = 6x$ $14 = 7x$ $2 = x$ $y = 3(2) = 6$	$3x = x - 2$ $2x = -2$ $x = -1$ $y = 3(-1) = -3$
corner point at (6, 4)	corner point at (2, 6)	corner pt. at (-1, -3)

So the corner points are (2, 6), (6, 4), and (-1, -3).

The maximum and minimum values of the optimization equation will always be on the corners of the feasibility region.

So, to find the solution put these three points into $Z = 3x + 4y$

i) (2, 6):	$z = 3(2) + 4(6) = 6 + 24 = 30$
ii) (6, 4):	$z = 3(6) + 4(4) = 18 + 16 = 34$
iii) (-1, -3):	$z = 3(-1) + 4(-3) = -3 - 12 = -15$

Then the maximum of $z = 34$ occurs at (6, 4), and the minimum of $z = -15$ occurs at (-1, -3).

There are two different general types of regions: bounded and unbounded regions. Bounded feasible regions have both a minimum and a maximum value. Unbounded feasible regions have either a minimum or maximum value, never

both. The minimum or maximum value of such objective functions always occurs at the vertex of the feasible region. This mathematical idea, however, is a proof that is for more advanced mathematics.

Applications of linear programming problem:-


1. Minimization of production cost & Maximization of profit .
2. Applications of Linear Programming in the Diet Problem
3. in real world problem Linear Programming is used etc.

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